Poisson measures for topological groups and their representations.

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1 Introduction

In articles [17, 18, 19, 23, 24, 25, 27, 29, 30, 35] Gaussian quasi-invariant measures on groups of diffeomorphisms and loop groups G relative to dense subgroups G' were constructed. In the non-Archimedean case the wider class of measures was investigated, than in the real case. The cases of Riemann and non-Archimedean manifolds were considered. There are few approaches for the construction of irreducible unitary representations. In articles [14, 18, 23, 24, 25] representations of dense subgroups G' associated with quasi-invariant measures on the entire groups were considered. In articles [12, 20, 21] irreducible representations of groups of diffeomorphisms $Diff(M)$ associated with measures on specific subsets of the unital type of products $M^{\mathbf{N}}$ of the manifolds M were investigated. In the publications [36, 38] irreducible unitary representations of groups of diffeomorphisms associated with real-valued Poisson measures on products of real manifolds were studied.

This article is related with unitary representations of G' associated with Poisson measures on $G^{\mathbf{N}}$ and uses quasi-invariant measures on G from the previous works. Several groups are considered: (1) (a) diffeomorphisms and (b) loop groups of real manifolds, (2) (a) diffeomorphisms and (b) loop groups of non-Archimedean manifolds over local fields. Besides these four cases

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further the fifth and the sixth cases are considered: for (3) (a) real and (b) non-Archimedean groups of diffeomorphisms $Diff(M)$ representations associated with Poisson measures on configuration spaces Γ_M contained in products of manifolds $M^{\mathbf{N}}$ are investigated. The case (3) (a) for real locally compact M was considered in [36, 38]. Here the cases of infinite-dimensional Banach manifold M (3) (a), non-Archimdean locally compact and non-locally compact Banach manifolds (3) (b) are investigated. For this quasi-invariant measures on M relative to $Diff(M)$ from [20, 21] are considered. Henceforth real-valued measures are considered. In §2 necessary Poisson measures are considered, definitions and notations are given. In §3 irreducible unitary representations are considered. Certainly not all results from [36, 38] can be transferred onto the cases considered here, moreover, there were necessary strong changes in many definitions, proofs and formulations of the theorems.

It is necessary to note that the theory of representations of non-locally compact groups differ substantially from that of locally compact groups. For example, irreducible unitary representations of locally compact Abelian groups are one-dimensional, that is, characters. But for non-locally compact Abelian groups there are infinite-dimensional irreducible unitary representations, which are even regular representations. It was shown in [1, 10] that there are infinite-dimensional topological vector spaces E and dense nuclear additive subgroups E' such that E' are linear subspaces and quasi-invariant measures μ on E relative to E' exist such that associated with them regular representations in the Hilbert space $L^2(E, \mu, \mathbf{C})$ are irreducible. The existence of such irreducible representations is even despite of the fact that projections μ_J of μ on one-dimensional subspaces J are equivalent with the Haar measures on J. This shows that non-locally compact case is more complicated than it may be supposed at the first glance. Also for definite groups G of diffeomorphisms and loops of definite real and non-Archimedean manifolds there are quasi-invariant measures μ on G relative to dense subgroups G' such that associated with them regular unitary representations are irreducible [14, 18, 23, 24, 25]. Such difference is caused by the existence of C^*-algebras associated with the Haar measures on locally compact groups [11], but no any C^*-algebra can be directly associated with a non-zero quasi-invariant measure on a non-locally compact group relative to a dense subgroup G'. Certainly, results on irreducibility of regular representations of infinite-dimensional topological groups G' depend stronlgy on quasi-invariant measures μ on G and a structure of G, where G' is dense in G.

2 Poisson measures.

2.1. Note. Let X denotes a manifold M for a group of diffeomorphisms $G = G(M)$ or the group G itself, where M is the C^∞-manifold over $\mathbf{R}$ or an analytic manifold over a local field and G is the loop group or the diffeomorphisms group as in the cited in §1 papers. Classes of smoothness of the groups G and G' are considered to be not less than C^1. The groups of diffeomorphisms G for the real C^∞-manifold M are denoted $Diff^t_{\beta,\gamma}(M)$ with $\infty \ge t \ge 1$, $\beta \ge 0$, $\gamma \ge 0$; the loop groups G for the real C^∞-manifold N are denoted $(L^mN)_{\gamma,Y}$ with $m+5 < \gamma \le \infty$, also another classes of smoothness and non-Archimedean groups and manifolds were considered (see theorem 3.4 [27] and also [6, 18, 23, 24, 21]). It was proved earlier, that G itself is the C^∞-manifold (in the case of the real group of diffeomorphisms for finite-dimensional Riemann manifolds M see also [2, 6]). Moreover, in the non-Archimedean case M and G have structures of the analytic manifolds with clopen disjoint charts. Clearly, G itself is not locally compact, since G considered as the manifold is infinite-dimensional over the corresponding field. When $X = M$ let us suppose, that X is the Banach non-compact manifold. In the non-Archimedean case it has embedding into the Banach space Z over the same local field $\mathbf{L}$ due to the partition of M into disjoint union of balls, so an atlas of M is supposed to be analytic and it has automatically foliated structure [22, 26]. In the real case it is supposed that M has a foliated structure with finite-dimensional submanifolds $M_n \subset M_{n+1}$ for each $n \in \mathbf{N}$ and $\bigcup_{n\in\mathbf{N}} M_n$ is dense in M, where $dim_{\mathbf{R}} M_n = k_n < \infty$ [21, 27, 30].

We remind the definition of the configuration space from [36] and also consider the ultrametric case of X. This means that a metric d in X satisfies the ultrametric inequality $d(x,y) \le \max(d(x,z), d(y,z))$ for each $x, y, z \in X$.

Let K be a complete separable metric space with a metric d, that is, X is a Polish space. In the ultrametric case this implies that its topological great inductive dimension is zero: $Ind(K) = 0$ [7]. Let $d^n_K(x,y) := \sum_{i=1}^n d(x_i,y_i)$ in the real case and $d(x,y) := \max_{1\le i\le n} d(x_i,y_i)$ in the non-Archimedean case be a metric in K^n, where $x = (x_i : i = 1,...,n) \in K^n$, $x_i \in K$. Put $\tilde{K}^n := (x \in K^n : x_i \ne x_j$ for each $i \ne j)$. Supply $\tilde{K}^n$ with a metric $\delta^n_K(x,y) := d^n_K(x,y)/[d^n_K(x,y) + d^n_K(x,(\tilde{K}^n)^c) + d^n_K(y,(\tilde{K}^n)^c)]$ in the real case and $\delta^n_K(x,y) := d^n_K(x,y)/[\max(d^n_K(x,y), d^n_K(x,(\tilde{K}^n)^c), d(y,(\tilde{K}^n)^c)]$ in the non-Archimedean case, where $A^c := K^n \setminus A$ for a subset $A \subset K^n$. Then $(\tilde{K}^n, \delta^n_K)$ is the Polish space. Moreover, if (K,d) is ultrametric, then

$(\tilde{K}^n, \delta^n_K)$ is ultrametric. Let also B^n_K denotes the collection of all n-point subsets of K. Then the metric δ^n_K is equivalent with the following metric $d^{(n)}_K(\gamma,\gamma') := \inf_{\sigma\in\Sigma_n} d^n_K((x_1,...,x_n),(x'_{\sigma(1)},...,x'_{\sigma(n)}))$, where Σ_n is the symmetric group of $(1,...,n)$, $\sigma \in \Sigma_n$, $\sigma : (1,...,n) \to (1,...,n)$; $\gamma,\gamma' \in B^n_K$. For each subset $A \subset K$ a number mapping $N_A : B^n_K \to \mathbf{N_o}$ is defined by the following formula: $N_A(\gamma) := card(\gamma \cap A)$, where $\mathbf{N} := \{1,2,3,...\}$, $\mathbf{N_o} := \{0,1,2,3,...\}$. Evidently, $d^{(n)}_K$ is the ultrametric, if d^n_K is the ultrametric. It remains to show, that δ^n_K is the ultrametric for the ultrametric space (K,d). For this we mention, that (i) $\delta^n_K(x,y) > 0$, when $x \ne y$, and $\delta^n_K(x,x) = 0$. (ii) $\delta^n_K(x,y) = \delta^n_K(y,x)$, since this symmetry is true for d^n_K and for [$*$] in the denumerator in the formula defining δ^n_K. To prove (iii) $\delta^n_K(x,y) \le \max(\delta^n_K(x,z),\delta^n_K(z,y))$ we consider the case $\delta^n_K(x,z) \ge \delta^n_K(y,z)$, hence it is sufficient to show, that $\delta^n_K(x,y) \le \delta^n_K(x,z)$. Let (a) $d^n_K(x,z) \ge \max(d^n_K(z,(\tilde{K}^n)^c), d^n_K(x,(\tilde{K}^n)^c))$, then $\delta^n_K(x,z) = 1$, hence $\delta^n_K(x,y) \le \delta^n_K(x,z)$, since $\delta^n_K(x,y) \le 1$ for each $x,y \in \tilde{K}^n$. Let (b) $d^n_K(x,(\tilde{K}^n)^c) > \max(d^n_K(x,z), d^n_K(z,(\tilde{K}^n)^c))$, then $\delta^n_K(x,z) = d^n_K(x,z)/d^n_K(x,(\tilde{K}^n)^c) \le 1$. Since $d^n_K(z,A) := \inf_{a\in A} d^n_K(z,a)$, then $d^n_K(z,(\tilde{K}^n)^c) \le \max(d^n_K(y,(\tilde{K}^n)^c), d^n_K(y,z))$. If $d^n_K(x,z) < d^n_K(z,(\tilde{K}^n)^c)$ and $d^n_K(x,y) \le d^n_K(x,z)$, then $d^n_K(z,(\tilde{K}^n)^c) \le d^n_K(x,(\tilde{K}^n)^c)$. Hence $d^n_K(x,y)\max(d^n_K(x,z), d^n_K(x,(\tilde{K}^n)^c), d^n_K(z,(\tilde{K}^n)^c)) \le d^n_K(x,z)\max(d^n_K(x,y), d^n_K(x,(\tilde{K}^n)^c), d^n_K(y,(\tilde{K}^n)^c))$. With the help of (ii) the remaining cases may be lightly written.

The Borel σ-field of B^n_K is denoted by $Bf(B^n_K)$. If $< S, \mathsf{L}, m >$ is the measure space, then its completion relative to m is denoted $Af(S,m)$, where S is a set, L is a σ-algebra of subsets of S, m is a real non-negative σ-finite measure on L. That is, the σ-algebra $Af(S,m)$ contains all subsets $A \subset B$ of $B \in \mathsf{L}$ for which $m(B) = 0$. In the non-Archimedean case the valuation group $\Gamma'_{\mathbf{L}} := \{|x|_{\mathbf{L}} : 0 \ne x \in \mathbf{L}\}$ of the local field $\mathbf{L}$ is discrete in $(0,\infty)$, hence subsets $U_\epsilon(y) := \{x \in K : d(x,y) < \epsilon\}$ are clopen (closed and open simultaneously) in $K = X$. Therefore, in the non-Archimedean case lemmas 1.1 and 1.2 from [36] have the following stronger forms.

2.2. Lemma. *For an ultrametric space (X,d) from §2.1 if U is a clopen set in X, then $\{\gamma : N_U(\gamma) \ge l\}$ is also clopen in X for each $l \in \mathbf{N_o}$.*

2.3. Lemma. *For an ultrametric space (X,d) from §2.1 and each $\epsilon > 0$ and each $\gamma \in B^n_X$ there exists a clopen subset $O_\epsilon(\gamma)$ which belongs to the smallest σ-algebra B for which functions N_B are measurable such that $\gamma \in O_\epsilon(\gamma) \subset \{\gamma' : d^{(n)}_X(\gamma,\gamma') < \epsilon\}$.*

Proof. For $\gamma = \{x_1, ..., x_n\}$ take $\eta \in \Gamma'_{\mathbf{L}}$ such that $\epsilon > \eta > 0$ and $U_{\eta p^{-n}}(x_i) \cap U_{\eta p^{-n}}(x_j) = \emptyset$ for each $i \ne j$. Put $O_\epsilon(\gamma) := \{\bigcap_{i=1}^n \{\gamma' : card(\gamma' \cap U_{\eta p^{-n}}(x_i)) \ge 1\}$, where $1 < p \in \Gamma_{\mathbf{L}}$, $p^{-1} = |\pi_{\mathbf{L}}|_{\mathbf{L}}$, $B(\mathbf{L}, 0, 1^-) = \pi_{\mathbf{L}} B(\mathbf{L}, 0, 1)$, $B(Y, x, r) := \{z \in Y : d_Y(x, z) \le r\}$, $B(Y, y, r^-) := \{z \in Y : d_Y(y, z) < r\}$ for an ultrametric space Y with an ultrametric d_Y.

2.4. Notes and definitions. Then theorems 1.1 and 1.2 from [36] are also true for all cases considered here. For this we mention, that as usually let $B_K := \bigoplus_{n=0}^\infty B_K^n$, where $B_K^0 := \{\emptyset\}$ is a singleton. Since X from §2.1 is not compact, then there exists an increasing sequence of subsets $K_n \subset X$ such that $X = \bigcup_n K_n$ and K_n are Polish spaces in the induced topology from X. Moreover, K_n can be chosen clopen in X in the non-Archimedean case. Then the following space $\Gamma_X := \{\gamma : \gamma \subset X$ and $card(\gamma \cap K_n) < \infty$ for each $n\}$ is called the configuration space and it is isomorphic with the projective limit $pr - \lim\{B_{K_n}, \pi_m^n, \mathbf{N}\}$, where $\pi_m^n(\gamma_m) = \gamma_n$ for each $m > n$ and $\gamma_n \in B_{K_n}$. If d_n denotes the metric in B_{K_n}, then $d_{n+1}|_{B_{K_n}} = d_n$, since $K_n \subset K_{n+1}$. Then $\prod_{n=1}^\infty B_{K_n} =: Y$ in the Tychonoff product topology is metrizable, that induces the metric in Γ_X. Moreover, in the non-Archimedean case the metric ρ in Y can be chosen satisfying the ultrametric inequality: $\rho(x, y) := d_n(x_n, y_n) p^{-n}$, where $n = n(x, y) := \min_{(x_j \ne y_j)} j$, $x = (x_j : j \in \mathbf{N}, x_j \in B_{K_j})$.

As it was proved in the papers cited in §1, on X from §2.1 there exist real measures m quasi-invariant relative to the left action of the corresponding group G' such that $m(K_n) < \infty$. In the case $X = G$, then G' is a dense subgroup in G. Quasi-invariance of m implies, that m are non-atomic. Let $K \in \{K_n : n \in \mathbf{N}\}$, then m_K denotes the restriction $m|_K$. Then $m_K^n := \bigotimes_{j=1}^n (m_K)_j$ is a measure on K^n and hence on $\tilde{K}^n$, since m are non-atomic, where $(m_K)_j = m_K$ for each j. Therefore, $P_{K,m} := exp(-m(K)) \sum_{n=0}^\infty m_{K,n}/n!$ is a probability measure on $Bf(B_K)$, where $m_{K,0}$ is a probability measure on the singleton B_K^0, and $m_{K,n}$ are images of m_K^n under the following mappings: $p_K^n : (x_1, ..., x_n) \in \tilde{K}^n \to \{x_1, ..., x_n\} \in B_K^n$. It was shown in §1.2 [36] that such system of measures $P_{K,n}$ is consistent, that is, $\pi_l^n P_{K_l,m} = P_{K_n,m}$ for each $n \le l$. This defines the unique measure P_m on $Bf(\Gamma_X)$, which is called the Poisson measure. For each $n_1, .., n_l \in \mathbf{N_o}$ and disjoint Borel subsets $B_1, ..., B_l$ in X there is the following equality:

(i) $P_m(\bigcap_{j=1}^l \{\gamma : card(\gamma \cap B_i) = n_i\}) = \prod_{i=1}^l m(B_i)^{n_i} exp(-m(B_i))/n_i!$.

The configuration space Γ_X consists of $\gamma \subset X$ such that $card(\gamma \cap K_n) < \aleph_0$ for each $n \in \mathbf{N}$. In the case of $Diff^t(M)$ this means that we need to consider such elements g of this group for which $supp(g) \subset K_n$ for some

$n \in \mathbf{N}$, for example, a subgroup with supports of its elements contained in the corresponding finite unions of charts, where $supp(g) := cl\{x \in M : g(x) \ne x\}$. Such subgroups are not Banach manifolds and they are denoted by $Diff_l(M)$. In the case of $X = G$ the initial configuration space Γ_X is not preserved by G', since there are $g \in G'$ such that gK_n is not contained in any K_m, because $supp(L_h) = G$ for each $e \ne h \in G'$, where $L_h g := hg$ denotes the left shift in G for $g, h \in G$.

Actually it is necessary to use more general construction in the case of $X = G$. Let $\tilde{\Gamma}_X := [\bigcup_{g \in G'} g\Gamma_X]/R$, where R is an equivalence relation: $(g\gamma)R(g'\gamma')$ if and only if $(g\gamma) = (g'\gamma')$, where $[\bigcup_{g \in G'} g\Gamma_X]$ is considered as the subset of $X^{\mathbf{N}}$. The group G' is separable, hence there exists a countable dense subset $\{g_j : j \in \mathbf{N}\}$. To each element $g \in G'$ there corresponds a subsequence $\{g_{j_n} : n \in \mathbf{N}\}$ converging to g in G'. Hence each $g\gamma$ is completely characterised by the corresponding subsequence $\{g_{j_n}\gamma : n \in \mathbf{N}\}$. Therefore, $\tilde{\Gamma}_X$ has the embedding into $X^{\mathbf{N}}$ as the closed subset, since the family of mappings $\{L_{g_j} : j \in \mathbf{N}\}$ separates points of $X^{\mathbf{N}}$ [7]. Hence $\tilde{\Gamma}_X$ is also metrizable and complete. The manifold $\tilde{\Gamma}_X$ for each its point has a neighbourhood diffeomorphic with the corresponding open subset of Γ_X, since for each K_n there exist a neighbourhood U'_n of e in G' and $m > n$ such that $U'_n K_n \subset K_m$. A choice of such sequence $K_n \subset Int(K_{n+1})$ with canonical closed subsets K_n is given independently in §2.9. The manifold $\tilde{\Gamma}_X$ is paracompact, consequently, it has a locally finite covering $\{S_l : l \in \mathbf{N}\}$, where S_l are open in $\tilde{\Gamma}_X$ and diffeomorphic with the corresponding open subsets Q_l of Γ_X for which $P_m(Q_l) < \infty$, $\zeta_l : S_l \to Q_l$ denote such diffeomorphisms. This means that the Poisson measure P_m on Γ_X induces the corresponding σ-additive σ-finite quasi-invariant relative to G' measure μ on $\tilde{\Gamma}_X$ such that $\mu(E) := C \sum_l P_m(\zeta_l(E \cap S_l))2^{-l}$ which is also denoted by P_m, where $E \in Bf(\tilde{\Gamma}_X)$, a constant $C > 0$ is chosen such that $\mu(\tilde{\Gamma}_X) = 1$. Therefore, P_m on $\tilde{\Gamma}_X$ is the probability measure as also for the case Γ_M for $dim_{\mathbf{L}} M < \infty$. This gives possibility to consider the case $X = G$ as well as the case $X = M$ for groups of diffeomorphisms $Diff^t(M)$ of class C^t with $1 \le t \le \infty$, which have structure of Banach manifolds from the papers cited above.

If the manifold M is locally compact and each K_n is chosen to be canonical closed compact subset, then for $Diff^t(M)$ the configuration spaces Γ_M and $\tilde{\Gamma}_M$ coincide. Indeed, if $\gamma \in \Gamma_M$, then $card(\gamma \cap K_n) < \aleph_0$ for each $n \in \mathbf{N}$. Each subset K_n is compact and canonically closed, hence is sequen-

tially compact [7]. This means that if $card((g\gamma)\cap K_l)=\aleph_0$ for some $l\in \mathbf{N}$ and $g\in Diff^t(M)$, then $\{g\gamma_j : j\in \mathbf{N}\}$ contains a convergent subsequence in K_l. But $\{\gamma_j : j\in \mathbf{N}\}=\gamma$ is the disrete subset of M, hence g^{-1} is not continuous, since $\{g\gamma_j : j\in \mathbf{N}\}$ is not closed in M. This contradicts supposition $g\in Diff^t(M)$, consequently, $g\gamma\in\Gamma_M$ for each $g\in Diff^t(M)$ for locally compact M and canonical closed compact subsets K_n in M. Therefore, $\bigcup_{g\in Diff^t(M)} g\Gamma_M=\Gamma_M$, since $g\Gamma_M\subset\Gamma_M$ for each $g\in Diff^t(M)$ and $e\Gamma_M=\Gamma_M$, consequently, $\tilde{\Gamma}_M=\Gamma_M$.

If M is not locally compact, for example, $M\setminus M_R=\bigcup_{j=1}^{\infty}\Omega_j$, where Ω_j are disjoint open subsets of M, $M_R:=\{x\in M : d_M(x,x_0)\le R\}$, $0<R<\infty$, x_0 is a fixed point in M and d_M is a metric in M, then there exists $g\in Diff^{\infty}(M)$ with $supp(g)$ bounded in M and a bounded infinite sequence of $\gamma_j\in M\setminus M_R$ which is discrete in M, that is, $cl\{\gamma_j : j\in \mathbf{N}\}=\{\gamma_j : j\in \mathbf{N}\}$, such that $card((g\gamma)\cap K_n)=\aleph_0$ for some canonically closed K_n in M, since each K_n is not locally compact, when $dim_{\mathbf{L}}M=\infty$, where $\mathbf{L}$ is the corresponding field either $\mathbf{R}$ or the local field. Hence in this case $\tilde{\Gamma}_M\ne\Gamma_M$.

If $X=G$, then in view of the choice of K_n in §2.9 that to fulfil demands on the measure m, there exists $g\in G'$ and $n\in\mathbf{N}$ such that gK_n is not contained in each K_l, where $l\in\mathbf{N}$. This g can be chosen by induction, since K_l are not locally compact for each l and G is not locally compact. Therefore, there exists a discrete infinite sequence γ in gK_n such that $card(\gamma\cap K_l)<\aleph_0$ for each $l\in\mathbf{N}$. But $\gamma\in\Gamma_G$ and $g^{-1}\in G'$ and $g^{-1}\gamma\in\tilde{\Gamma}_G\setminus\Gamma_G$, since $card((g^{-1}\gamma)\cap K_n)=\aleph_0$. Hence $\tilde{\Gamma}_G\ne\Gamma_G$ in this case also.

The group G' and X and Γ_X have structures of the C^{∞}-manifolds, since X is the C^{∞}-manifold. Therefore, $\tilde{\Gamma}_X$ is the C^{∞}-manifold also. In the non-Archimedean case M, G', G and hence Γ_X and $\tilde{\Gamma}_X$ are analytic manifolds with disjoint clopen charts, since $\Gamma'_{\mathbf{L}}$ is discrete in $(0,\infty)$ and Γ_X and $\tilde{\Gamma}_X$ are infinite-dimensional over $\mathbf{L}$ [22].

It is necessary to note, that for $X=G$ the dense subgroup G' acts by the left shifts $L_h : G\to G$ as the diffeomorphism for each $h\in G'$, where G is either the loop group or the diffeomorphisms group. Therefore, lemmas 2.1, 2.2 and theorems 2.1, 2.2 and 2.3 from [36] are applicable to the cases considered here, since Γ_X produces charts for $\tilde{\Gamma}_X$ and P_m on Γ_X induces P_m on $\tilde{\Gamma}_X$. Theorem 2.3 from [36] can be applied to the real and non-Archimedean cases of $X=M$.

2.5. Definition. (see [9] §19.3.) Let G' be a group acting from the left on the measure space $< X, \mathsf{L}, m >$. Then $< X, \mathsf{L}, m >$ is called a measure G'-transformation space if (i) $xW \in \mathsf{L}$ whenever $x \in G'$ and $W \in \mathsf{L}$, and (ii) $m(xW) = 0$ whenever $x \in G'$, $W \in \mathsf{L}$ and $m(W) = 0$.

2.6. Note. For the considered here cases and $Bf(X) \subset \mathsf{L}$ conditions of definition 2.5 are fulfilled for the quasi-invariant measure m on X relative to G'.

2.7. Definition. The measure G'-transformation space $< S, \mathsf{L}, m >$ is ergodic under G' if, whenever, $V, W \in \mathsf{L}$ with $m(V)m(W) \ne 0$, there exists $x \in G'$ such that $m(xV \cap W) \ne 0$.

2.8. Note. It was proved in [18, 20, 21, 23, 24, 25] that m on X is ergodic under G' for the considered here cases $(1-3)$, since m is quasi-invariant relative to G'. In cases $(1, 2)$ at first m was constructed on a neighbourhood W of e in G. But theorem 2.3 from [36] can not be applied to the cases $X = G$ for the probability measure m on X, since in view of the construction of the Gaussian measure m on G there are $\epsilon > 0$ and $n \in \mathbf{N}$ such that for each $\psi \in G'$ with $\psi(K_n) \cap K_n = \emptyset$ the following integral is rather large: $\int_G |\rho_m^{1/2}(\psi, x) - 1| m(dx) > \epsilon$, where $m^\psi(E) := m(\psi^{-1}E)$ for each $E \in Af(X, m)$, $\rho_m(\psi, x) := m^\psi(dx)/m(dx)$.

There are locally finite coverings $\{g_jW_j : j \in \mathbf{N_o}\}$ of G and $\{g_jW'_j : j \in \mathbf{N_o}\}$ of G', since G and G' are paracompact spaces relative to their own topologies τ and τ' respectively and G' is dense in G, where $W_0 = W$, $W_j \subset W$ for each j, $W' \subset W \cap G'$, $W'_0 = W'$, $W'_j \subset W'$ for each j, $g_j \in G'$ for each j, $g_0 = e$, W_j are open in G and W'_j are open in G'. Analogously for the pair G' and $X = M$ in cases $(3)(a, b)$. Then m on W can be extended as a σ-finite measure on $Bf(G)$ by the formula:

(i) $m(V) := \sum_{j=0}^\infty m(g_j^{-1}(V \cap g_jW_j))$, since $0 < m(W) < \infty$. The group G is not locally compact, hence $m(G) = \infty$. Using analogous procedure with a locally finite covering $\{g_jW_j : j \in \mathbf{N_o}\}$ with W_j open in M and a neighbourhood W of a marked point $x_0 \in M$ without relation between W' and W we get a σ-finite measure m on M for non-locally compact manifold M with $m(M) = \infty$. We choose in these cases $m(K_n) < \infty$ for each $n \in \mathbf{N}$. As follows from the cited papers it is possible to choose $K_n \subset \bigcup_{j=0}^n g_jW_j$ and m such that

(ii) for each $\epsilon > 0$ and each $n \in \mathbf{N}$ there exists $\psi \in G'$ such that $\psi(K_n) \cap K_n = \emptyset$ and $\bigcup_n K_n = X$ and $\int_X |\rho_m^{1/2}(\psi, x) - 1|^2 m(dx) < \epsilon$. Then it is proved below in theorem 2.9 that such m exists and P_m on $\tilde{\Gamma}_X$ is ergodic.

Henceforth, such σ-finite measures m on X are used with $m(X) = \infty$, since for $m(X) = 1$ the corresponding measures P_m are not ergodic (see note after definiton 1 in §2 [36]).

2.9. Theorem. *There exist quasi-invariant σ-finite measures m on X relative to the groups G' with $m(X) = \infty$ satisfying condition (ii) from §2.8. For such m the Poisson measure P_m on $\tilde{\Gamma}_X$ is ergodic .*

Proof. To prove P_m is ergodic on Γ_X we use the fact, that m is ergodic on X. The measure space $< S, \mathsf{L}, m >$ is said to have property (P) if, for any locally m-measurable subset W of S such that $xW \ominus W$ is locally m-null for each $x \in G'$, either W is locally m-null or $S \setminus W$ is locally m-null. The measure space $< S, \mathsf{L}, m >$ is called parabounded if there exists a pairwise disjoint subfamily W of L such that (i) for each $A \in \mathsf{L}$, $\{B \in \mathsf{W} : A \cap B \ne \emptyset\}$ is countable, and (ii) $X \setminus \bigcup_{W \in \mathsf{W}} W$ is locally m-null. It was proved in proposition 19.5 [9] that if $< S, \mathsf{L}, m >$ is ergodic it has property (P). Conversely, if $< S, \mathsf{L}, m >$ has property (P) and is parabounded, it is ergodic. The space Γ_X is isomorphic with the projective limit $pr - \lim\{B_{K_n}, \pi^n_m, \mathbf{N}\}$, which is the closed subset in $\prod_n B_{K_n}$. The latter is the Polish space, hence $\tilde{\Gamma}_X$ is the Polish space [7]. The measure spaces $< X, Af(X, m), m >$ and $< \tilde{\Gamma}_X, Af(\tilde{\Gamma}_X, P_m), P_m >$ are parabounded, since X and $\tilde{\Gamma}_X$ are the Polish spaces and hence are the Radon spaces (see chapter 1 in [5]), that is, the class of compact subsets approximates from below the corresponding measures $m|_{K_n}$ and P_m. Therefore, it remains to show, that $< \tilde{\Gamma}_X, Af(\tilde{\Gamma}_X, P_m), P_m >$ has property (P). But this follows from theorem 2.3 [36] and §2.8, if to show that condition 2.8 (ii) is fulfilled for m and $U'_n K_n \subset Int(K_{n+1})$ for the corresponding K_n in X and neighbourhoods U'_n of e in G', since there are the local diffeomorphisms $\zeta_l : S_l \to Q_l$ from §2.4 and P_m and m are σ-finite measures. In this situation integral equalities and inequalities from the proof of theorem 2.3 [36] are transferable onto the case of $\tilde{\Gamma}_X$ considered here.

For the construction of such m take it at first on an open subset $U \subset X$ such that W is sufficiently small: $W'W \subset U$. In the case of $G = X$ in addition let $e \in U$ and $U^{-1} = U$, $W^{-1} = W$, $W'^{-1} = W'$ (see references in §2.8). The quasi-invariance factor $\rho_m(x, y)$ is continuos on $W' \times W$ and $\rho_m(e, y) = 1$, where $\rho_m(x, y) := m^x(dy)/m(dy)$, $m^x(A) := m(x^{-1}A)$ for each $x \in G'$ and $A \in Af(W, m)$. Take open subsets $W_0 \subset W$ and $W'_0 \subset W'$ for which $|\rho_m(x, y) - 1| < 1$ for each $(x, y) \in W'_0 \times W_0$.

The measure m is regular and approximated from above by the class of open subsets [5, 9]. Therefore, it is possible to choose by induction open

subsets $W_j \subset W_0$ and $e \in W'_j \subset W'_0$ and a sequence of elements $g_j \in G'$ such that $m(g_j^{-1}(g_jW_j \cap [\bigcup_{i=1}^{j-1} g_iW_i])) < 2^{-j}$ and $|\rho_w(x,y) - 1| < 2^{-j}$ on $g_jW'_j \times g_jW_j$, where w is a measure on $Bf(g_jW_j)$ defined by the following formula $w(g_jA) := m(A)$ for each $A \in Bf(W_j)$, $g_0 = e$. Then m on $Bf(X)$ is defined by formula 2.8(i) and certainly has the extension m onto $Af(X, m)$.

The measure m is induced from the corresponding measure λ on the Banach space Y due to the local diffeomorphism $A : U \to V$, where V is an open neighbourhood of 0 in Y and U is open in X. From the quasi-invariance of λ relative to shifts from a dense subspace Y' it follows a property:

(α) for each Borel subset $E \subset Y$ which is a C^1-submanifold in Y of codimension 1 in Y (over the field $\mathbf{R}$ or the non-Archimedean local field) such that T_yE is not subset of Y' for each $y \in E$ it follows that $\lambda(E) = 0$, since λ is the quasi-invariant non-negative σ-additive and σ-finite measure. In particular, for finite-dimensional $X = M$ over the corresponding field the space Y is finite-dimensional and λ can be taken as the Haar measure on Y (in the real case it concides with the Lebesgue measure). For infinite-dimensional real X, particularly for $X = G$, the measure λ can be taken Gaussian. For infinite-dimensional X over the local field the wider class of measures λ was constructed in the papers cited in §1. Then we choose (take) by induction a sequence $K_n \subset \bigcup_{i=0}^{n} g_iW_i$ satisfying the following conditions ${U'}_nK_n \subset Int(K_{n+1})$ for each n with $\bigcup_n K_n = X$ and $m(K_n \setminus Int(K_n)) = 0$ and K_n are canonical closed subsets, that is, $cl(Int(K_n)) = K_n$, since m is quasi-invariant and has not any atoms and due to property (α) of λ, where $cl(A)$ denotes the closure of a subset $A \subset X$ in X, $Int(A)$ denotes the interior of A in X, ${U'}_n$ are the corresponding (open) neighbourhoods of e in G' such that ${U'}_n \subset W'$. The space X is Polish, hence each K_n is the Polish topological subspace [7]. Certainly, in the non-Archimedean cases each K_n can be chosen clopen (closed and open) in X, that is, $Int(K_n) = K_n = cl(K_n)$, since the base of the topology of X consists of clopen subsets. Since X is not locally compact, then there exists the sequence $\{K_n : n \in \mathbf{N}\}$ fulfilling condition 2.8(ii).

2.10. Note. In cases (3)(a, b) for $X = M$ and $G' = Diff^t(M)$ in addition we have the following.

2.11. Definition. Let ${G'}_{K_n} := \{\psi \in G' : \psi|_{K_n^c} = id\}$ and let f be a symmetric measurable function defined on $\tilde{K}_n^l$, where $l \in \mathbf{N}$, $A^c := X \setminus A$ for a subset A in X, K_n are canonical closed subsets with $\bigcup_n K_n = X$ and $K_n \subset$

K_{n+1} and $m(K_n \setminus Int(K_n)) = 0$ for each $n \in \mathbf{N}$. In the non-Archimedean case let also K_n be clopen in M, which automatically implies $K_n \setminus Int(K_n) = \emptyset$. The measure m is called ${G'}^l_{K_n}$-ergodic, if f is constant modulo null sets, then $f(x_1, ..., x_l) = f(\psi(x_1), ..., \psi(x_l))$ for $m^l_{K_n}$-a.e $x = (x_1, ..., x_l)$ for all $\psi \in G'$.

2.12. Theorem. *If for each n the measure m is ${G'}^l_{K_n}$-ergodic for some $N \geq n$ and all l, then P_m is G'-ergodic.*

Proof. As it was shown in papers [18, 21, 27, 29, 30] the subgroups ${G'}_{K_n}$ are correctly defined for canonical closed subsets K_n in $M = X$, ${G'}_{K_n} \subset G'$ for each n, since from $\psi|_{K^c_n} = id$ it follows, that $\psi|_{cl(K^c_n)} = id$. The rest of the proof is as in the proof of theorem 2.4 [36], which can be applied locally and then with the help of the local diffeomorphisms $\zeta_l : S_l \to Q_l$ is extendable onto the case of $\tilde{\Gamma}_X$ considered here, since ${G'}^l_{K_N} \tilde{K}^l_N = \tilde{K}^l_N$ and for the measure $\nu(A) := P_m(E \cap A)$ for each $A \in Bf(\tilde{\Gamma}_X)$ we have $\nu(B) = \int_0^\infty P_{cm}(B)\lambda(dc)$ for each $B \in Bf(\Gamma_X)$, where $c \geq 0$ and λ is a suitable Borel measure on $[0, \infty)$. From $P_m(\zeta_l(A \cap S_l)) = 0$ for each l it follows, that $P_m(A) = 0$. Thus if $\lambda(\{1\}) > 0$, then $P_m(A) = 0$; if $\lambda(\{1\}) = 0$, then $P_m(A^c) = 0$, where A is a P_m-measurable subset of $\tilde{\Gamma}_X$ for which $P_m(A \triangle \psi^{-1}A) = 0$ for all $\psi \in G'$, where $A \triangle B := (A \setminus B) \cup (B \setminus A))$.

2.13. Note. From theorem 2.12 it can be deduced in another way, than it was done in theorem 2.9, that P_m on $\tilde{\Gamma}_X$ is G'-ergodic in cases $(3)(a, b)$ for $X = M$, when m and K_n are chosen in accordance with §2.8 and §2.11. The proof of this is analogous to that of theorem 2.5 [36], since m is ergodic and quasi-invariant with the continuous quasi-invariance factor $\rho_m(\psi, x)$ on $G' \times X$, $m(X) = \infty$ and $m(K_n \setminus Int(K_n)) = 0$, since due to §2.4 there are the local diffeomorphisms $\zeta_l : S_l \to Q_l$ and $Diff^t(X; K)\tilde{K}^l = \tilde{K}^l$ for each canonical closed subset K in X, where $Diff^t(X; K) := \{f \in Diff^t(X) : f|_{K^c} = id\}$.

2.14. Lemma. *Let Y be a canonically closed subset in X, $Y \subset K_n$ for some $n \in \mathbf{N}$. Suppose that μ is a quasi-invariant measure on $\tilde{\Gamma}_X$ relative to $G' = Diff^t(X)$ for a C^∞-manifold $X = M$ (in the non-Archimedean case an analytic manifold M) and μ_n be a restriction of μ on $B^n_Y \times \tilde{\Gamma}_{X \setminus Y}$ and μ'_n and $\mu"_n$ be projections of μ_n on B^n_Y and $\tilde{\Gamma}_{X \setminus Y}$ respectively. Then μ_n is equivalent with $\mu'_n \times \mu"_n$. In the non-Archimedean case this is also true for Y clopen in X.*

Proof. In view of §2.9 $m(Y \setminus Int(Y)) = 0$. The group $Diff^t(X; X \setminus Y)$ is a closed subgroup of $Diff^t(X)$, hence $\tilde{\Gamma}_{X \setminus Y} = (Diff^t(X; X \setminus Y)\Gamma_X)/R$

is a C^∞-submanifold of $\tilde{\Gamma}_X$ (see also §2.4). The measures $\mu'_n \times \mu"_n$ with μ_n are equivalent if and only if μ and $\mu' \times \mu"$ are equivalent, since μ is quasi-invariant relative to G' and non-atomic, where μ' is a projection of $\mu|_{B_Y \times \tilde{\Gamma}_{X \setminus Y}}$ on B_Y and $\mu"$ is a projection of μ on $\tilde{\Gamma}_{X \setminus Y}$. On the other hand, $G' B^n_X = B^n_X$ for each $n \in \mathbf{N}$ and $G' \tilde{\Gamma}_X = \tilde{\Gamma}_X$, also $Diff^t(X; X \setminus Y) \tilde{\Gamma}_{X \setminus Y} = \tilde{\Gamma}_{X \setminus Y}$. On the other hand, $B^n_Y \times \tilde{\Gamma}_{X \setminus Y}$ is the Borel subset of $\tilde{\Gamma}_X$, since B^n_Y is the Borel subset of Γ_X. For the rest of the proof are necessary two propositions.

2.15. Proposition. *In the group $Diff^t(Int(Y))$ there exists a countable family of one-parameter subgroups G_l such that generated by them group $J \subset Diff^t(Y)$ acts transitively on B^n_Y.*

Proof. For $Diff^t(Int(Y))$ one-parameter subgroups can be chosen as in proposition 2.1 [38] with the help of [6] and theorems about existence of one-parameter subgroups of $Diff^t(Y)$ for infinite-dimensional Banach manifolds M from [17, 21, 26, 27], where one-parameter subgroups are real for real M and g^b with $b \in \mathbf{L}$ for M over the local field $\mathbf{L}$ such that $g^a g^b = g^{a+b}$ for each $a, b \in \mathbf{L}$. In the non-Archimedean case one-parameter subgroups can aslo be indexed by $b \in B(\mathbf{L}, 0, 1)$, where $B(S, x, r) := \{y \in S : d_S(x, y) \le r\}$ denotes a ball in a metric space S with a metric d_S and a point $x \in S$. This is possible, since M and $T_x M$ are separable spaces for each $x \in M$ and using countable atlas $At(M) = \{(U_j, \phi_j) : j\}$ of M and considering one-parameter subgroups with $supp(g^b) \subset U_j$ for each $b \in \mathbf{L}$ for the corresponding chart U_j, where either $\mathbf{L} = \mathbf{R}$ or $\mathbf{L}$ is the local field, U_j are open in M and $\phi_j : U_j \to V_j$ are diffeomorphisms, V_j are open in the corresponding Banach space.

2.16. Proposition. *Let $\mathbf{L}$ or may be $B(\mathbf{L}, 0, 1)$ also in the non-Archimedean case acts measurably in a measure space $(M, Bf(M), \mu)$ such that μ is quasi-invarint relative to the action of $\mathbf{L}$ or $B(\mathbf{L}, 0, 1)$ on M, where M is a C^∞-manifold (analytic in the non-Archimedean case) and μ is induced by a quasi-invariant non-negative σ-additive and σ-finite measure η relative to shifts from a dense subspace Z' and η is on the Borel field $Bf(Z)$ of the separable Banach space Z over a field $\mathbf{L}$ which is either $\mathbf{L} = \mathbf{R}$ or a local field such that $Z = T_x M$ for each $x \in M$. Suppose that a partition ζ of M is invariant by $mod(\mu)$ relative to the action of $\mathbf{L}$ or $B(\mathbf{L}, 0, 1)$ on M and projections of η onto one-dimensional over $\mathbf{L}$ subspaces are equivalent with the non-negative Haar measure λ on $\mathbf{L}$. Then for μ-almost each $C \in \zeta$ the conditional measures μ^C are quasi-invariant relative to the action of $\mathbf{L}$ or $B(\mathbf{L}, 0, 1)$ respectively.*

Proof. The proof is almost the same as in proposition 2.2 [38] with the substitution of $\mathbf{R}$ onto $\mathbf{L}$ or may be $B(\mathbf{L},0,1)$ in the non-Archimedean case and using the Haar measure λ on a locally compact subgroup S of $\mathbf{L}$ with $\lambda(\mathbf{L}\setminus S)=0$ or $\lambda(B(\mathbf{L},0,1)\setminus S)=0$, which implies $S=\mathbf{L}$ or $S=B(\mathbf{L},0,1)$ respectively by the A. Weil theorem, since each quasi-invariant measure on a locally compact group (relative to its action on itself) is equivalent with the Haar measure [4].

Continuation of the proof of lemma 2.14. In view of proposition 2.15 there exists a subgroup J which acts transitively on $B^n_{Int(Y)}$. In view of proposition 2.16 from an isomorphism of one-parameter subgroup G_l with $\mathbf{L}$ or $B(\mathbf{L},0,1)$ for $\mu"_n$-a.e. configurations $\gamma\in\tilde{\Gamma}_{X\setminus Y}$ the conditional measure μ^γ_n on B^n_Y is quasi-invariant relative to each one-parameter subgroup G_l, hence realtive to the minimal subgroup J of $Diff^t(X)$ generated by $\bigcup_{l=1}^\infty G_l$. The measure μ^γ_n on B^n_Y induces the measure η on $T_{\gamma^n}B^n_Y$ for each $\gamma^n\in B^n_Y$. This measure η is completely characterised by its finite-dimensional projections η_n onto subspaces F_n such that $F_n\subset F_{n+1}$ for each $n\in\mathbf{N}$ and $\bigcup_n F_n$ is dense in $T_{\gamma^n}B^n_Y$ (see about weak distributions [28, 37]). It is supposed that the manifold M has the foliated structure such that $M_n\subset M_{n+1}$ and $dim_{\mathbf{L}}M_n=k_n<\infty$ for each $n\in\mathbf{N}$ and $\bigcup_{n\in\mathbf{N}}M_n$ is dense in M. Theorefore, to μ^γ_n there corresponds a family of measures $\tilde{\eta}_n$ on M_n with the help of a locally finite coverings and the exponential mapping $exp:\tilde{M}\to M$ from the neighbourhood $\tilde{T}M$ of M in TM onto M such that $exp_x:V_x\to W_x$ are local diffeomorphisms of open subsets V_x in T_xM and W_x in M with $x\in M$. A measure $\tilde{\eta}_n$ is quasi-invariant relative to $J_n:=\{g\in J: g_{M\setminus M_n}=id\}$. The manifold M_n is locally compact, hence $\tilde{\eta}_n$ is equivalent with the Riemann volume element on M_n in the real case and with the restriction of the Haar measure from $\mathbf{L}^{k_n}$ onto M_n in the non-Archimedean case, since in the latter case M_n is embeddable into $\mathbf{L}^{k_n}$ due to a partition of M_n into a disjoint union of balls. In view of the Kakutani theorem II.4.1 [5] μ_n is equivalent with $\mu'_n\times\mu"_n$, since a finite measure ζ on $Bf(A\times B)$ is equivalent with the direct product $\zeta_A\times\zeta_B$, where ζ_A and ζ_B are projections of ζ on Hausdorff topological spaces A and B respectively.

3 Unitary representations associated with the Poisson measures.

3.1. Definitions and notes. Let $H := L^2(\tilde{\Gamma}_X, P_m, \mathbf{C})$ be the standard Hilbert space of equivalence classes of measurable functions $f : \tilde{\Gamma}_X \to \mathbf{C}$ for which $\|f\|^2_H := \int_{\tilde{\Gamma}_X} |f(x)|^2 P_m(dx) < \infty$, where P_m is the Poisson measure given in §2.4. Then consider the following representation:

(*i*) $U_m(\psi)f(\gamma) := \rho^{1/2}_{P_m}(\psi,\gamma)f(\psi^{-1}(\gamma))$,

where $\rho_{P_m}(\psi,\gamma) := P_{m^\psi}(d\gamma)/P_m(d\gamma)$, $\gamma \in \tilde{\Gamma}_X$, $f \in H$, $\psi \in G'$, $m^\psi(E) := m(\psi^{-1}E)$ for each $E \in Af(X,m)$. That is, $U_m : G' \to U(H)$, where $U(H)$ is the unitary group of the Hilbert space H. The topology of $U(H)$ is induced by the operator norm in the space $L(H)$ of bounded linear operators $S : H \to H$, $d(A,B) := d(B^{-1}A, I) := \|B^{-1}A - I\|_{L(H)}$ is the metric in $U(H)$, where $A, B \in U(H)$, I denotes the unit operator on H.

In cases $(3)(a,b)$ of $X = M$ and $G' = Diff^t(M)$ these representations can be generalised with the help of the symmetric group Σ_n representations in the following manner, where Σ_n is the group of all (bijective) automorphisms σ of the set $\{1,2,..,n\}$ with $n \in \mathbf{N}$ and Σ^∞ is the symmetric group of $\mathbf{N}$ (that is, of all bijective mappings of $\mathbf{N}$). Let $q : \Sigma_n \to U(W)$ be the unitary representation of Σ_n, where W is the Hilbert space, or $q : \Sigma^\infty \to U(W)$. Then $s_n : B^n_X \to \tilde{X}^n$ or $s : \tilde{\Gamma}_X \to \tilde{X}^\infty$ produces a mapping $\sigma : G' \times B^n_X \to \Sigma_n$ or $s : G' \times \tilde{\Gamma}_X \to \Sigma^\infty$ by the formula $s_n(\psi^{-1}(\gamma)) = \psi^{-1}(s_n(\gamma))\sigma(\psi,\gamma)$ or $s(\psi^{-1}(\gamma)) = \psi^{-1}(s(\gamma))\sigma(\psi,\gamma)$, where s_n is a measurable cross-section of $p_n : \tilde{X}^n \to B^n_X$ and s of $p : \tilde{X}^\infty \to \tilde{\Gamma}_X$ such that $p_n(x_1,...,x_n) = \{x_1,...,x_n\}$ for $n \in \mathbf{N}$ or $p(x_1,x_2,...) = \{x_1,x_2,...\}$, $(x_1,...,x_n)\sigma = (x_{\sigma(1)},...,x_{\sigma(n)})$ or $(x_1,x_2,...)\sigma = (x_{\sigma(1)},x_{\sigma(2)},...)$ respectively. Then with each pair (n,q) or (∞,q) is associated a unitary representation of G' in $L^2(B^n_X, m^n, W)$ or in $L^2(\tilde{\Gamma}_X, P_m, W)$ respectively such that

(*ii*) $V^q_m(\psi)f(\gamma) := \rho^{1/2}_{m^n}(\psi,\gamma)q(\sigma(\psi,\gamma))f(\psi^{-1}(\gamma))$, or

(*iii*) $V^q_m(\psi)f(\gamma) := \rho^{1/2}_{P_m}(\psi,\gamma)q(\sigma(\psi,\gamma))f(\psi^{-1}(\gamma))$,

where m^n is the image measure of the direct product of n copies of m by the map p_n and $\rho_{m^n}(\psi,\gamma) := (m^n)^\psi(d\gamma)/m^n(d\gamma)$, $(m^n)^\psi(E) := m^n(\psi^{-1}E)$ for each $E \in Af(B^n_X, m^n)$, $\psi \in G'$, $\gamma \in B^n_X$. As usually the space $L^2(B^n_X, m^n, W)$ denotes the space of equivalence classes of measurable functions $f : B^n_X \to W$ for which $\|f\|^2_{L^2(B^n_X,m^n,W)} := \int_{B^n_X} \|f(x)\|^2_W m^n(dx) < \infty$. Then U_m and V^q_m

define new representation $U^q_m := U_m \otimes V^q_m$.

3.2. Note. For the group of diffeomorphisms of the real finite-dimensional manifold M such representations were defined in [38], where it was mentioned that the representations V^q_m are in a weak respect analogous to the construction of H. Weyl for the classical Lie groups. For $W = \{0\}$ and $q = I$ the representation V^q_m is evidently irreducible [12, 21, 20], hence as in theorem 1.1 [38] for the considered here cases we have, that (i) if q is the irreducible representation of Σ_n with $n \in \mathbf{N}$, then V^q_m is the irreducible representation of the diffeomorphism group G'; (ii) $V^{q_1}_m$ and $V^{q_2}_m$ are equivalent if and only if $n_1 = n_2$ and q_1 of S_{n_1} is equivalent to q_2 of S_{n_2}.

3.3. Note. Let $X = M$ be a finite-dimensional over a local field $\mathbf{K}$ non-compact manifold embedded as an open subset into $\mathbf{K^n}$. Suppose that m is the restriction $m = \lambda|_M$ of the Haar measure λ on $\mathbf{K^n}$ normalised by $\lambda(B(\mathbf{K^n}, 0, 1)) = 1$. Let $Diff^t(X, m)$ denotes the subgroup of $G' = Diff^t(X)$ of the non-Archimedean class of smoothness C^t such that $\rho_m(\psi, x) = 1$ for each $\psi \in Diff^t(X, m)$ and $x \in X$, where $1 \le t \le \infty$.

3.4. Theorem. *Let X and $Diff^t(X, m)$ be the same as in §3.3. Then the restriction of the representation V^q_m from §3.1 on $Diff^t(X, m)$ is irreducible.*

Proof. For finite-dimensional M over the local field $\mathbf{L}$ there is the equality $\tilde{\Gamma}_M = \Gamma_M$ due to §2.4. Since diffeomorphisms ψ with locally linear $(\psi - id)$ are contained in $Diff^t(X, m)$, for example, when $\|\psi - id\|_{C^1(X \to \mathbf{K^n})} < 1$. Then for each pairwise distinct points $x_1, ..., x_n \in X$ there are neighbourhoods $O_1, ..., O_n$ such that their closures $\bar{O}_j$ are C^1-diffeomorphic with balls in $\mathbf{K^n}$ and $\bar{O}_i \cap \bar{O}_j = \emptyset$ for each $i \ne j$ and $m(O_1) = ... = m(O_n)$. Moreover, for each transposition $(k_1, ..., k_n)$ of $(1, ..., n)$ there exists a diffeomorphism $\psi \in Diff^t(X, m)$ with $\psi(\bar{O}_i) = \bar{O}_{k_i}$. Such ψ exists due to partition of X into disjoint union of sufficiently small clopen balls U_l such that $\bar{O}_j = O_j$ for each $j = 1, ..., n$ and for each j there exists l_j such that $O_j = U_{l_j}$ while $\psi(O_j) = O_{k_j}$ and diameters of all O_j are equal to each other. Having ψ on $\bigcup_{j=1}^n O_j =: E$, it is possible to extend ψ as id on $X \setminus E$.

Let now Y be a clopen compact submanifold of X and consider subspace $\tilde{L}^2(Y, m, \mathbf{C})$ consisting of $f \in L^2(Y, m, \mathbf{C})$ with $\int_Y f(y) m(dy) = 0$. Let $H_1 \ne \{0\}$ be an invariant subspace of $\tilde{L}^2(Y, m, \mathbf{C})$ relative to the regular representation $(U_\psi f)(y) := f(\psi^{-1} y)$ of $Diff^t(Y, m)$. For each ball O in Y there exists $f \in H_1$ such that $f \ne 0$ and $supp(f) \subset O$. Further analogously to the proof of lemma 2 from §1 [38] we get, that such representation of

$Diff^t(Y,m)$ is irreducible. If O_j with $j=1,...,n$ are clopen balls in X, then a subgroup $G^0(O_1,...,O_n;X)$ of elements $\psi \in Diff^t(X,m)$ with $\psi|_{O_j}=id$ for each $j=1,...,n$ acts trivially on $(\bigotimes_{j=1}^n L^2(O_j,m,\mathbf{C}))\otimes W$. Then quite analogously to the proof of theorem 1.2 [38] we get the statement of this theorem.

3.5. Note. Let $\tilde{\mathbf{N}}^n := \{a=(i_1,...,i_n) | i_j \ne i_s \text{ for each } i\ne s\}$, $l_2(\tilde{\mathbf{N}}^n,W) := \{\phi | \phi : \tilde{\mathbf{N}}^n \to W$, such that $\|\phi\|^2 := \sum_{a\in\tilde{\mathbf{N}}^n}\|\phi(a)\|_W^2 < \infty\}$ and $H^q := \{\phi \in l_2(\tilde{\mathbf{N}}^n,W) | \phi(i_{\sigma(1)},...,i_{\sigma(n)}) = q^{-1}(\sigma)\phi(i_1,...,i_n))$ for all $\sigma\in S_n\}$, where q is a unitary representation of S_n in a Hilbert space W. In the case $X=M$ the representation q and W may be non-trivial with $n\in\mathbf{N}$, for $X=G$ and G' acting on X we set $q=I$ and $W=\{0\}$ and $n=0$. We denote by Σ^∞ the set of all permutations (bijections) of $\mathbf{N}$ and put $\sigma a = (\sigma(i_1),...,\sigma(i_n))$ for $\sigma\in\Sigma^\infty$ and $a\in\tilde{\mathbf{N}}^n$. Then a function $\sigma : G'\times\tilde{\Gamma}_X \to \Sigma^\infty$ is defined by the formula $s(\psi^{-1}(\gamma)) = \psi^{-1}(s(\gamma))\sigma(\psi,\gamma)$, where s is a measurable (admissible) cross section of the map $p : \tilde{X}^\infty \ni (x_1,x_2,...) \mapsto \{x_1,x_2,...\}\in\tilde{\Gamma}_X$ posessing the following property:

(α) if $card(\gamma\cap X_1)=k_1$, $card(\gamma\cap(K_2\setminus K_1))=k_2$,..., $card(\gamma\cap(K_n\setminus K_{n-1}))=k_n$,..., then the first k_1 elements of $s(\gamma)$ are in $\gamma\cap K_1$, the next k_2 of $s(\gamma)$ are in $\gamma\cap(K_2\setminus K_1)$ and so on. For $X=M$ finite-dimensional over $\mathbf{L}$ and $\psi\in Diff^t(K_l) := \{\psi\in Diff(X) : \psi|_{K_l^c}=id\}$, then $\sigma(\psi,\gamma)\in\Sigma_r$, so H^q is non-trivial in general for this case, where $K_l^c := X\setminus K_l$. The latter property in general may be untrue for infinite-dimensional manifold M or for $X=G$ and G' acting on G, therefore, we consider $q=I$ and $n=0$ and $W=\{0\}$ for $X=G$. For infinite-dimensional $X=M$ over $\mathbf{L}$ let us drop condition (α) and let q be a representation of Σ^∞ in $U(H^q)$, where H^q is defined analogously with the H^q for n but with the substitution of n onto ∞ and Σ_n onto Σ^∞.

Then there exists the following unitary representation of G' in the space $L^2(\tilde{\Gamma}_X,P_m,\mathbf{C})\otimes H^q$ (which is isomorphic with $L^2(\tilde{\Gamma}_X,P_m,\mathbf{C})$ for $X=G$):

(i) $U^q_m(\psi)F(\gamma,a) := \rho^{1/2}_{P_m}(\psi,\gamma)F(\psi^{-1}(\gamma),\sigma(\psi,\gamma)^{-1}a)$ (see ρ_{P_m} in §3.1).

3.6. Theorem. *The representations from §3.1 and §3.5 in the case $X=G$ are equivalent, in the case of finite-dimensional $X=M$ over $\mathbf{L}$ for the group of diffeomorphism acting on M the representation $U_m\otimes V^q_m$ is equivalent with U^q_{nom}, where q is a representation of the symmetric group Σ_n.*

Proof. In the case $X=G$ this follows from their definitions, that is, U_m given by formula 3.1.(i) is equivalent with U^q_m, since $q=I$ and W is trivial.

In the case of the C^∞-manifold $X = M$, which is finite-dimensional over $\mathbf{R}$, it was proved in theorem 3.2 [38]. In the non-Archimedean case the proof is analogous, but instead of differentiabilty of measures their pseudodifferentiability should be considered as in [28]. In view of proposition 2.16 the quasi-invariant measure m on M relative to $Diff^t(M)$ is equivalent with the restriction of the Haar measure λ for $\mathbf{L}^k$ on M, that is, $\lambda|_M \sim m$, where M is embedded into $\mathbf{L}^k$ for the corresponding local field $\mathbf{L}$. In view of §2.4 $\Gamma_M = \tilde{\Gamma}_M$ for locally compact M. Therefore, convolutions of measures $\mu_1 * \mu_2$ are correctly defined on Γ_M as an image of the product measure $\mu_1 \times \mu_2$ on $\Gamma_M \times \Gamma_M$ relative to a mapping $(\gamma_1, \gamma_2) \mapsto (\gamma_1 \cup \gamma_2)$ from $\Gamma_M \times \Gamma_M$ to Γ_M. Then $n \circ \mu$ denotes $\mu * m_n$, where m_n is a measure on B^n_X corresponding to the restriction $P_m|_{B^n_X}$, $n_1 \circ (n_2 \circ \mu)$ is equivalent with $(n_1 + n_2) \circ \mu$ for each n_1 and $n_2 \in \mathbf{N}$, $0 \circ \mu$ is equivalent with μ. For each $\psi \in Diff^t(M)$ we have $\psi(\mu_1 * \mu_2) = (\psi\mu_1) * (\psi\mu_2)$, where $t \geq 1$ and $\psi\mu := \mu^\psi$, $\mu^\psi(E) := \mu(\psi^{-1}E)$ for each $E \in Bf(M)$. Therefore, for each pair μ_1 and μ_2 of quasi-invariant measures, their convolution is also a quasi-invaraint measure. Then all necessary results from §2.3-5 of [38] can lightly be transferred onto the non-Archimedean case.

3.7. Note. In the papers [16, 27, 30, 35] quasi-invariant measures on the diffeomorphisms groups of real Banach manifolds were constructed. Purely Gaussian measures quasi-invariant relative to dense subgroups were constructed in the cases of Euclidean and Hilbertian at infinity manifolds and also for definite closed subgroups $Diff^t_k(M) := \{f \in Diff^t(M) : (\Delta^j f)|_{\partial M} = (\Delta^j) id|_{\partial M}$ for each $j = 0, 1, ..., k\}$ and $Dif^t_l(M) := \{f \in Diff^t(M) : (\partial^j_\nu f)|_{\partial M} = (\partial^j_\nu id)|_{\partial M}$ for each $j = 0, 1, ..., l\}$ of $Diff^t(M)$ for compact C^s-manifolds M with a boundary $\partial M \neq \emptyset$, where $Diff^t(M)$ has a class of smoothness by Hölder C^t, also a class of smoothness H^t by Sobolev or Besov was considered for $t > dim_{\mathbf{R}} M + 5$, Δ denotes the Beltrami-Laplace operator on M, ∂_ν denotes the partial differentiation along normal to the boundary in local coordinates, $\Delta^0 = I$ and $\partial^0 = I$ are the unit operators. In particular for a compact manifold with the boundary purely Gaussian measures μ on $Diff^t_k(M)$ and $Dif^t_l(M)$ quasi-invariant relative to dense subgroups $Diff^{t'}_k(M)$ and $Dif^{t"}_l(M)$ were constructed, where k and l and $t' - t > 0$ and $t" - t > 0$ are dependent on $dim_{\mathbf{R}} M$, $s > t' + 2$ and $s > t" + 2$ respectively. The cases of Schwarz class of smoothness also were considered. The given below theorem in the real case for finite measures was proved shorlty earlier in [25], in the non-Archimedean case it is contained in [18].

In theorem 3.8 a quasi-invariant σ-finite σ-additive measure is considered, which may be unbounded. The cases of σ-finite non-negative measures and probability measures on G are considered quite analogously. Certainly this theorem is applicable not only to Gaussian measures but also to measures which have definite properties of the quasi-invariance factors ρ_μ such that a family of continuos functions $\{\rho_\mu^{1/2}(z,g)=\phi(g): z\in G'\}$ parametrized with $z\in G'$ separates points of G (see more precisely the proof below). It is essential in the proof that G is the infinite-dimensional non-locally group and G' is its dense subgroup such that the measure μ is ergodic. Evidently, if μ' is a measure equivalent with μ, then the regular representations associated with them are equivalent due to the isomorphism $f(g)\mapsto(\mu'(dg)/\mu(dg))^{1/2}f(g)$ of the Hilbert space $L^2(G,\mu',\mathbf{C})$ with $L^2(G,\mu,\mathbf{C})$, where $f\in L^2(G,\mu',\mathbf{C})$ and $g\in G$.

3.8. Theorem. *Let G be a group of diffeomorphisms with a real probability quasi-invariant measure μ relative to a dense subgroup G' as in §3.7. Then μ may be chosen such that the associated regular unitary representation of G' is irreducible.*

Proof. Let a measure ν on a Banach space H be of the same type as in the proofs of theorems in papers cited in §3.7 such that a local diffeomorphism $A: W\to V_H$ induces a quasi-invaraint measure on W and then with the help of left shifts $g_j\in G'$ on the entire group G, where W is an open neighbourhood of e in G and V_H is an open neighbourhood of 0 in H. We choose a constant multiplier $c>0$ for μ such that $c\mu(W)=1$ and then denote such normalized measure by μ. The measure μ on G is σ-finite, since $0<\mu(W)<\infty$ and G is with a countable base and a locally finite covering as in §2.8 and §2.9. A strong continuity of the regular representation $T: G'\to U(L^2(G,\mu,\mathbf{C}))$ follows from the continuity of the quasi-invariance factor $\rho_\mu(\psi,x)$ by $(\psi,x)\in G'\times G$ and the embedding $T_eG'\hookrightarrow T_eG$ of trace class, where $T^\mu:=T$, $T(z)f(g):=\rho_\mu^{1/2}(z,g)f(z^{-1}g)$, $z\in G'$, $g\in G$, $f\in L^2(G,\mu,\mathbf{C})$. Let a ν-measurable function $f: H\to\mathbf{C}$ be such that $\nu(\{x\in H: f(x+y)\ne f(x)\}=0$ for each $y\in X_0$ with $f\in L^1(H,\nu,\mathbf{C})$. Let also $P_k: l_2\to L(k)$ be projectors such that $P_k(x)=x_k$ for each $x=(\sum_{j\in\mathbf{N}}x^je_j)$, where $x_k:=\sum_{j=1}^k x^je_j$, $x_k\in L(k)$, $L(k):=sp_{\mathbf{R}}(e_1,...,e_k)$, $sp_{\mathbf{R}}(e_j: j\in\mathbf{N}):=\{y: y\in l_2; y=\sum_{j=1}^n x^je_j; x^j\in\mathbf{R}; n\in\mathbf{N}\}$. Since the dense subspace X in H is isomorphic with l_2, then each finite-dimensional subspace $L(k)$ is complemented in H [32]. From the proof of Proposition II.3.1 [5] in view of the Fubini Theorem there exists a sequence of cylindrical

functions $f_k(x) = f_k(x^k) = \int_{H\ominus L(k)} f(P_k x + (I - P_k)y)\nu_{I-P_k}(dy)$ which converges to f in $L^1(H, \nu, \mathbf{C})$, where $\nu = \nu_{L(k)} \otimes \nu_{I-P_k}$, ν_{I-P_k} is the measure on $H \ominus L(k)$. Each cylindrical function f_k is ν-almost everywhere constant on H, since $L(k) \subset X_o$ for each $k \in \mathbf{N}$, consequently, f is ν-almost everywhere constant on H. From the construction of G' and μ with the help of the local diffeomorphism A and ν it follows that, if a function $f \in L^1(G, \mu, \mathbf{C})$ satisfies the following condition $f^h(g) = f(g)\ (mod\ \mu)$ by $g \in G$ for each $h \in G'$, then $f(x) = const\ (mod\ \mu)$, where $f^h(g) := f(hg)$, $g \in G$.

Let $f(g) = Ch_U(g)$ be the characteristic function of a subset U, $U \subset G$, $U \in Af(G, \mu)$, then $f(hg) = 1 \Leftrightarrow g \in h^{-1}U$. If $f^h(g) = f(g)$ is true by $g \in G$ μ-almost everywhere, then $\mu(\{g \in G : f^h(g) \ne f(g)\}) = 0$, that is $\mu((h^{-1}U) \triangle U) = 0$, consequently, the measure μ satisfies the condition (P) from §VIII.19.5 [9], where $A \triangle B := (A \setminus B) \cup (B \setminus A)$ for each $A, B \subset G$. For each subset $E \subset g_j W_j$ with $g_j \in G'$ and $W_j \subset W$ from §2.9 the outer measure is bounded, $\mu^*(E) \le 1$, since $\mu(W) = 1$ and μ is non-negative [4], consequently, there exists $F \in Bf(G)$ such that $F \supset E$ and $\mu(F) = \mu^*(E)$. This F may be interpreted as the least upper bound in $Bf(G)$ relative to the latter equality. In view of the Proposition VIII.19.5 [9] the measure μ is ergodic, that is for each $U \in Af(G, \mu)$ and $F \in Af(G, \mu)$ with $\mu(U) \times \mu(F) \ne 0$ there exists $h \in G'$ such that $\mu((h \circ E) \cap F) \ne 0$.

From Theorem I.1.2 [5] it follows that $(G, Bf(G))$ is a Radon space, since G is separable and complete. Therefore, a class of compact subsets approximates from below each measure μ^f, $\mu^f(dg) := |f(g)|\mu(dg)$, where $f \in L^2(G, \mu, \mathbf{C}) =: \bar{H}$. Due to the Egorov Theorem 2.3.7 [8] for each $\epsilon > 0$ and for each sequence $f_n(g)$ converging to $f(g)$ for μ-almost every $g \in G$, when $n \to \infty$, there exists a compact subset K in G such that $\mu(G \setminus \mathsf{K}) < \epsilon$ and $f_n(g)$ converges on K uniformly by $g \in \mathsf{K}$, when $n \to \infty$. In each Hilbert space $L^2(\mathbf{R^n}, \lambda, \mathbf{R})$ the linear span of functions functions $f(x) = exp[(b, x) - (ax, x)]$ is dense, where b and $x \in \mathbf{R^n}$, a is a symmetric positive definite real $n \times n$ matrix, $(*, *)$ is the standard scalar product in $\mathbf{R^n}$ and λ is the Lebesgue measure on $\mathbf{R^n}$. If a non-linear operator U on X satisfies conditions of Theorem 26.1 [37], then $\nu^U(dx)/\nu(dx) = |detU'(U^{-1}(x))|\rho_\nu(x - U^{-1}(x), x)$, where $\nu^U(B) := \nu(U^{-1}B)$ for each $B \in Bf(X)$, $\rho_\nu(z, x) = exp\{\sum_{l=1}^{\infty}[2(z, e_l)(x, e_l) - (z, e_l)^2]/\lambda_l\}$ by Theorem 26.2 [37], where λ_l and e_l are eigenvalues and eigenfunctions of the correlation operator J on X enumerated by $l \in \mathbf{N}$, $z \in X_0$, $\rho_\nu(z, x) := \nu_z(dx)/\nu(dx)$, $\nu_z(B) := \nu(B - z)$ for each $B \in Bf(X)$. Since the

Gaussian measure ν induces with the help of subalgebras of cylinder subsets in $Bf(H)$ and $Bf(X)$ the corresponding Gaussian measure on H, which is also denoted by ν, then analogous formulas of quasi-invariance factor are true for ν on H [5]. Hence in view of the Stone-Weierstrass Theorem A.8 [9] an algebra $\mathsf{V}(Q)$ of finite pointwise products of functions from the following space $sp_{\mathbf{C}}\{\psi(g) := (\rho(h,g))^{1/2} : h \in G'\} =: Q$ is dense in $L^2(G,\mu,\mathbf{C})$, since $\rho(e,g) = 1$ for each $g \in G$ and $L_h : G \to G$ are diffeomorphisms of the manifold G, $L_h(g) = hg$.

For each $m \in \mathbf{N}$ there are C^∞-curves ϕ^b_j, where $j = 1,...,m$ and $b \in (-2,2) := \{a : a \in \mathbf{R}; -2 < a < 2\}$ is a parameter, such that $\phi^b_j|_{b=0} = e$ and $(\partial\phi^b_j/\partial b)|_{b=0}$ are linearly independent in T_eG' vectors and $\phi_j := \phi^1_j$, $\phi_j \in G' \cap W$, $j = 1,...,m$, since G' is the infinite-dimensional group, which is complete relative to its own uniformity. Then the following condition $det(\Psi(g)) = 0$ defines a submanifold G_Ψ in G of codimension over $\mathbf{R}$,

(i) $codim_{\mathbf{R}} G_\Psi \ge 1$, where $\Psi(g)$ is a matrix dependent from $g \in G$ with matrix elements $\Psi_{l,j}(g) := D^{2l}_{\phi_j}(\rho(\phi_j,g))^{1/2}$. If $f \in \bar{H}$ is such that $(f(g),(\rho(\phi,g))^{1/2})_{\bar{H}} = 0$ for each $\phi \in G' \cap W$, then differentials of these scalar products by ϕ are zero. But $\mathsf{V}(Q)$ is dense in $\bar{H}$ and in view of condition (i) this means that $f = 0$, since for each m there are $\phi_j \in G' \cap W$ such that $det\Psi(g) \ne 0$ μ-almost everywhere on G, $g \in G$. If $\|f\|_{\bar{H}} > 0$, then $\mu(supp(f)) > 0$, consequently, $\mu((G'supp(f)) \cap W) = 1$, since $G'U = G$ for each open U in G and for each $\epsilon > 0$ there exists an open U, $U \supset supp(f)$, such that $\mu(U \setminus supp(f)) < \epsilon$.

This means that the linear span over $\mathbf{C}$: $sp_{\mathbf{C}}\{Ch_{g_kW_k}\phi(g) : \phi(g) = \rho^{1/2}_\mu(h,g); h \in G'\}$ is dense in $L^2(g_kW_k,\mu,\mathbf{C})$. Therefore, the following vector $f_0(g) := \sum_{j=0}^\infty 2^{-j}Ch_{g_jW_j}(g)$ is cyclic for T^μ, since $\{g_jW_j : j \in \mathbf{N_o}\}$ is a locally finite covering and $\tilde{\mu}(dg) = f_0(g)\mu(dg)$ is a finite measure with continuous $\rho_{\tilde{\mu}}$ such that $f(g) \mapsto f_0^{1/2}(g)f(g)$ establishes isomorphism of $L^2(G,\tilde{\mu},\mathbf{C})$ with $L^2(G,\mu,\mathbf{C})$. If $f_k \in L^\infty(g_kW_k,\mu,\mathbf{C})$ for each $k \in \mathbf{N}$ and $f_k|_{(g_kW_k \cap g_lW_l)} = f_l|_{(g_kW_k \cap g_lW_l)}$ for each $g_kW_k \cap g_lW_l \ne \emptyset$ and $\sup_k \|f_k\|_{L^\infty(g_kW_k,\mu,\mathbf{C})} < \infty$, then there exists $f \in L^\infty(G,\mu,\mathbf{C})$ such that $f|_{g_kW_k} = f_k$ for each $k \in \mathbf{N}$, where $Ch_W(g)$ is the characteristic function of W, that is, $Ch_W(g) = 1$ for each $g \in W$ and $Ch_W(g) = 0$ for each $g \in G \setminus W$. From the construction of μ it follows that for each $f_{1,j}$ and $f_{2,j} \in \bar{H}$, $j = 1,...,n$, $n \in \mathbf{N}$ and each $\epsilon > 0$ there exists $h \in G'$ such that $|(T_hf_{1,j},f_{2,j})_{\bar{H}}| \le \epsilon|(f_{1,j},f_{2,j})_{\bar{H}}|$, when $|(f_{1,j},f_{2,j})_{\bar{H}}| > 0$, since G is the Radon space by Theorem I.1.2 [5] and G

is not locally compact. This means that there is not any finite-dimensional G'-invariant subspace H' in $\bar{H}$ such that $T_h H' \subset H'$ for each $h \in G'$ and $H' \neq \{0\}$. Hence if there is a G'-invariant closed subspace H' in $\bar{H}$ it is isomorphic with the subspace $L^2(V, \mu, \mathbf{C})$, where $V \in Bf(G)$.

Let A_G denotes a $*$-subalgebra of $L(\bar{H}, \bar{H}) = L(\bar{H})$ generated by the family of unitary operators $\{T_h : h \in G'\}$. In view of the von Neumann double commuter Theorem (see §VI.24.2 [9]) A_G" coincides with the weak and strong operator closures of A_G in $L(\bar{H}, \bar{H})$, where $\mathsf{A}_G{}'$ denotes the commuting algebra of A_G and $\mathsf{A}_G" = (\mathsf{A}_G{}')'$. Suppose that λ is a probability Radon measure on G' such that λ has not any atoms and $supp(\lambda) = G'$. Let $a(x) \in L^\infty(G, \mu, \mathbf{C})$, f and $g \in \bar{H}$, $\beta(h) \in L^2(G', \lambda, \mathbf{C})$. Since $L^2(G', \lambda, \mathbf{C})$ is infinite-dimensional, then for each finite family of $a \in \{a_1, ..., a_m\} \subset L^\infty(G, \mu, \mathbf{C})$, $f \in \{f_1, ..., f_m\} \subset \bar{H}$ there exists $\beta(h) \in L^2(G', \lambda, \mathbf{C})$, $h \in G'$, such that β is orthogonal to $\int_G \bar{f}_s(g)[f_j(h^{-1}g)(\rho(h, g))^{1/2} - f_j(g)]\mu(dg)$ for each $s, j = 1, ..., m$. Hence each operator of multiplication on $a_j(g)$ belongs to A_G", since there exists $\beta(h)$ such that $(f_s, a_j f_l) = \int_G \int_{G'} \bar{f}_s(g)\beta(h)(\rho(h, g))^{1/2} f_l(h^{-1}g)\lambda(dh)\mu(dg)$ $= \int_G \int_{G'} \bar{f}_s(g)\beta(h)(T_h f_l(g))\lambda(dh)\mu(dg)$, $\int_G \bar{f}_s(g)a_j(g)f_l(g)\mu(dg) = \int_G \int_{G'} \bar{f}_s(g)$ $\beta(h)\ f_l(g)\lambda(dh)\mu(dg) = (f_s, a_j f_l)$. Hence A_G" contains subalgebra of all operators of multiplication on functions from $L^\infty(G, \mu, \mathbf{C})$.

Let us remind the following. A Banach bundle B over a Hausdorff space G' is a bundle $< B, \pi >$ over G', together with operations and norms making each fiber B_h ($h \in G'$) into a Banach space such that conditions $BB(i - iv)$ are satisfied: $BB(i)$ $x \to \|x\|$ is continuous on B to $\mathbf{R}$; $BB(ii)$ the operation $+$ is continuous as a function on $\{(x, y) \in B \times B : \pi(x) = \pi(y)\}$ to B; $BB(iii)$ for each $\lambda \in \mathbf{C}$, the map $x \to \lambda x$ is continuous on B to B; $BB(iv)$ if $h \in G'$ and $\{x_i\}$ is any net of elements of B such that $\|x_i\| \to 0$ and $\pi(x_i) \to h$ in G', then $x_i \to 0_h$ in B, where $\pi : B \to G'$ is a bundle projection, $B_h := \pi^{-1}(h)$ is the fiber over h (see §II.13.4 [9]). If G' is a Hausdorff topological group, then a Banach algebraic bundle over G' is a Banach bundle $\mathsf{B} =< B, \pi >$ over G' together with a binary operation $\bullet$ on B satisfying conditions $AB(i - v)$: $AB(i)$ $\pi(b \bullet c) = \pi(b)\pi(c)$ for b and $c \in B$; $AB(ii)$ for each x and $y \in G'$ the product $\bullet$ is bilinear on $B_x \times B_y$ to B_{xy}; $AB(iii)$ the product $\bullet$ on B is associative; $AB(iv)$ $\|b \bullet c\| \le \|b\| \times \|c\|$ $(b, c \in B)$; $AB(v)$ the map $\bullet$ is continuous on $B \times B$ to B (see §VIII.2.2 [9]). With G' and a Banach algebra A the trivial Banach bundle $\mathsf{B} = \mathsf{A} \times G'$ is associative, in particular let $\mathsf{A} = \mathbf{C}$ (see §VIII.2.7 [9]).

The regular representation T of G' gives rise to a canonical regular $\bar{H}$-

projection-valued measure $\bar{P}$: $\bar{P}(W)f = Ch_W f$, where $f \in \bar{H}$, $W \in Bf(G)$, Ch_W is the characteristic function of W. Therefore, $T_h\bar{P}(W) = \bar{P}(h \circ W)T_h$ for each $h \in G'$ and $W \in Bf(G)$, since $\rho(h, h^{-1} \circ g)\rho(h, g) = 1 = \rho(e, g)$ for each $(h, g) \in G' \times G$, $Ch_W(h^{-1} \circ g) = Ch_{h \circ W}(g)$ and $T_h(\bar{P}(W)f(g)) = \rho(h, g)^{1/2}\bar{P}(h \circ W)f(h^{-1} \circ g)$. Thus $< T, \bar{P} >$ is a system of imprimitivity for G' over G, which is denoted T^μ. This means that conditions $SI(i - iii)$ are satisfied: $SI(i)$ T is a unitary representation of G'; $SI(ii)$ $\bar{P}$ is a regular $\bar{H}$-projection-valued Borel measure on G and $SI(iii)$ $T_h\bar{P}(W) = \bar{P}(h \circ W)T_h$ for all $h \in G'$ and $W \in Bf(G)$.

For each $F \in L^\infty(G, \mu, \mathbf{C})$ let $\bar{\alpha}_F$ be the operator in $L(\bar{H}, \bar{H}) = L(\bar{H})$ consisting of multiplication by F: $\bar{\alpha}_F(f) = Ff$, $f \in \bar{H}$. The map $F \to \bar{\alpha}_F$ is an isometric $*$-isomorphism of $L^\infty(G, \mu, \mathbf{C})$ into $L(\bar{H}, \bar{H})$ (see §VIII.19.2[9]). Therefore, Propositions VIII.19.2,5[9] (using the approach of this particular case given above) are applicable in our situation.

If $\bar{p}$ is a projection onto a closed T^μ-stable subspace of $\bar{H}$, then $\bar{p}$ commutes with all $\bar{P}(W)$. Hence $\bar{p}$ commutes with multiplication by all $F \in L^\infty(G, \mu, \mathbf{C})$, so by VIII.19.2 [9] $\bar{p} = \bar{P}(V)$, where $V \in Bf(G)$. Also $\bar{p}$ commutes with all T_h, $h \in G'$, consequently, $(h \circ V) \setminus V$ and $(h^{-1} \circ V) \setminus V$ are μ-null for each $h \in G'$, hence $\mu((h \circ V) \triangle V) = 0$ for all $h \in G'$. In view of ergodicity of μ and proposition VIII.19.5 [9] either $\mu(V) = 0$ or $\mu(G \setminus V) = 0$, hence either $\bar{p} = 0$ or $\bar{p} = I$, where I is the unit operator. Hence T is the irreducible unitary representation.

Almost analogous proof was done in the case of loop groups with the corresponding quasi-invaraint measures and with the use of the spectral theorem for the family of commuting unitary operators, since the loop group is Abelian in [23, 24]. In the non-Archimedean case G' has the analytic atlas $At(G') = \{(U_j, \psi_j) : j \in \mathbf{N}\}$ with disjoint clopen charts, hence curves ϕ_j^b can be chosen locally analytic with a restriction on the corresponding neighbourhood U_1 of e being analytic, where $b \in \mathbf{L}$. Substitution of differentiation on pseudodifferentiation along ϕ_j^b by parameter $b \in B(\mathbf{L}, 0, 1)$ produces by formula $det(\Psi(g)) = 0$ an analytic submanifold G_Ψ in G with $codim_{\mathbf{L}} G_\Psi \geq 1$, since G is the analytic manifold.

3.9. Theorem. *Let P_m be the ergodic Poisson measure on $\tilde{\Gamma}_X$ as in §2.4, 2.9 and q be an irreducible representation of the symmetric group Σ_n ($q = I$ for $X = G$ and may be non-trivial for $X = M$ finite-dimensional over the coresponding field $\mathbf{L}$ and a group of diffeomorphisms G' of M). Then the representation $U_m \otimes V_m^q$ from §3.1 is irreducible.*

Proof. The case of real finite-dimensional M was proved in [38]. The case of non-Archimedean M with $dim_{\mathbf{L}}M < \infty$ follows from §3.5, since $\tilde{\Gamma}_X = \Gamma_X$ in this case. Indeed, $U_m \otimes V^q_m$ is equivalent with $U^q_{n\circ m}$ and $f\phi \in L^2(\Gamma_X, P_m, \mathbf{C})$, if $f \in L^2(\Gamma_X, P_m, \mathbf{C})$ and $\phi \in L^\infty(\Gamma_X, P_m, \mathbf{C})$. Then each subspace L in $L^2(\Gamma_X, P_m, \mathbf{C})\otimes H^q$ invariant relative to $G' = Diff^t(X)$ is also invariant relative to multiplications on functions $\phi \in L^\infty(\Gamma_X, P_m, \mathbf{C})$, since $\mathsf{L} = \bigoplus_{r,i} \mathsf{L}^i_{l,r}$, where $\mathsf{L}^i_{l,r} := \mathsf{L}\cap(L^2(B^r_{K_l}\times\Gamma_{X\setminus K_l}, \mu_r, \mathbf{C})\otimes W^i_r\otimes C^i_r)$ are subspaces invariant relative to $Diff^t(K_l)$, $\mu = P_m$ and μ_r is the corresponding measure on $B^r_{K_l} \times \Gamma_{X\setminus K_L}$. In view of lemma 2.14 the measure μ_n is equivalent with $\mu'_n \times \mu"_n$ and further as at the end of §3 [38].

The remaining cases are proved analogously to the proof of theorem 3.8 (and see [18, 23, 24]) applied to the pair $(G', \tilde{\Gamma}_X)$ instead of (G', G), since $\tilde{\Gamma}_X$ is C^∞-manifold and from infinite differentiability or pseudodifferentiability of m it follows, that P_m is also infinite differentiable or pseudodifferentiable respectively, morever, P_m is the ergodic measure due to theorem 2.9. In the case of $X = M$ the measures on X are chosen to be such that $sp_{\mathbf{C}}\{\rho^{1/2}_m(z, x) = \phi(x) : z \in G'\}$ is dense in $L^2(X, m, \mathbf{C})$ in accordance with §2.9 and the cited papers there, for example, Gaussian measures or product measures of special type on T_xM induce the demanded measures on M, where $x \in M$.

It remains only to establish that the density ρ_{P_m} has the demanded properties. For this it is necessary to use the fact that operators L_h on X (either $X = M$ or $X = G$) are infinitely strongly differentiable by $h \in G'$ and there exists a dense subset $G"$ in G' such that $(L_h)^{(n)} \ne 0$ for each $n \in \mathbf{N}$ and each $h \in G"$. Therefore, $(AL_hA^{-1})^{(n)} \ne 0$ for each $h \in G"$, where $A : U \to V_H$ is a local diffeomorphism, where U and V_H are open subsets in X and the corresponding Banach space H respectively as in §2.9, §3.8 and the cited above papers. In the Hilbert space $L^2(\mathbf{R^{mn}}, \lambda, \mathbf{C})$ is dense the following linear span $sp_{\mathbf{C}}\{exp[\sum_{l=1}^k (a^l_1, x^l) - (a^l_2x^l, x^l)] =: \phi(y)|a^l_1 \in \mathbf{R^m}$, $a^l_2 \in \mathbf{R^m}$, $\sum_{l=1}^k a^l_{2,j} > 0$ for each $j = 1, ..., m$; $a^l_{2,j} \ge 0$ for each $j = 1, ..., m$, $l = 1, ..., k$; and if $a^l_{1,j} \ne 0$, then there exists l' such that $2l' > l$ and $a^{l'}_{2,j} > 0$, $a^l_i := (a^l_{1,i}, ..., a^l_{i,m})$, $a^l_{i,j} \in \mathbf{R}$, $x^l_j := S_l(y_{j,1}, ..., y_{j,n})$, $l = 1, ..., k$, $j = 1, ..., m\}$, where $k = k(m, n) \in \mathbf{N}$ is chosen such that $z \mapsto \{S_l(z) : l = 1, .., k\}$ is a bijection of $\mathbf{R^n}$, $z = (z_1, ..., z_n) \in \mathbf{R^n}$, $S_l(z) := \sum_{i=1}^n (z_i)^l$ is a power sum of degree l, $(*, *)$ is a scalar product in $\mathbf{R^m}$, λ_{mn} is a Lebsgue measure on $\mathbf{R^{mn}}$. For the local field $\mathbf{L}$ in the Hilbert space $L^2(\mathbf{L^{mn}}, \nu, \mathbf{C})$ is dense the following linear span $sp_{\mathbf{C}}\{exp[\sum_{l=1}^k -|(a^l, (b^l + x^l)|^2] =: \phi(y)|a^l \in \mathbf{L^m}$, $b^l \in \mathbf{L^m}$, $\sum_{l=1}^k |a^l_j| > 0$

for each $j = 1, ..., m$, $a^l := (a^l_1, ..., a^l_m)$, $a^l_j \in \mathbf{L}$, $x^l_j := S_l(y_{j,1}, ..., y_{j,n})$, $l = 1, ..., k$, $j = 1, ..., m\}$, where $k = k(m, n) \in \mathbf{N}$ is chosen such that $z \mapsto \{S_l(z) : l = 1, .., k\}$ is a bijection of $\mathbf{L^n}$, $z = (z_1, ..., z_n) \in \mathbf{L^n}$, $S_l(z) := \sum_{i=1}^n (z_i)^l$ is a power sum of degree l, $(z, q) := \sum_{i=1}^n z_i q_i$, $q \in \mathbf{L^n}$, ν_{mn} is the Haar measure on $\mathbf{L^{mn}}$.

Using charts in B^n_X we get projections $L^2(B^n_X, m^n, \mathbf{C})$ into $L^2(\mathbf{R^{mn}}, \lambda_{mn}, \mathbf{C})$ in the real case and into $L^2(\mathbf{L^{mn}}, \nu_{mn}, \mathbf{C})$ in the non-Archimedean case. Then we use Taylor expansion up to $o(d^{k'+1}_{G'}(\psi, h))$ of L_h in a suitable neighbourhoods hU' in G' of elements $h \in G$" with U' open in G' with $e \in U' \subset W'$ and with $k' = 2k$ in the real case and $k' = k$ in the non-Archimedean case, where $d_{G'}$ is the metric in G' in its own uniformity, $\psi \in hU'$. For a manifold $C^t(M, N)$ of mappings $f : M \to \mathbf{N}$ of class of smoothness C^t with $t \ge 1$ for C^∞-manifolds M and N in the real case and analytic manifolds in the non-Archimedean case the tangent manifold $TC^t(M, N)$ is isomorphic with $C^t(M, TN)$ and for the n-th order we get $T^n C^t(M, N) = C^t(M, T^n N)$ (see also [6, 13, 24]). Then $T^n Diff^t(M)$ is a submanifold in $C^t(M, T^n M)$. Let $C^t(M, m_0; N, n_0)$ be a family of mappings $f \in C^t(M, N)$ preserving marked points $f(m_0) = n_0$, $m_0 \in M$ and $n_0 \in N$, where in the real case $M = S^m$ is the m-dimensional real sphere and $dim_{\mathbf{R}} N > m$. Analogously for others classes of smoothness ω considered for construction of loop groups $L(M, m_0; N, n_0)_\omega$, elements of which are closures of orbits $cl\{f(\psi(x)) : \psi \in G(M), \psi(m_0) = m_0\}$, where $G(M)$ denotes the group of diffeomorphisms of M of the corresponding class of smoothness and with certain additional construction in the non-Archimedean case [19, 23, 24]. Hence the manifold $T^n L(M, m_0; N, n_0)_\omega$ is isomorphic with the following manifold $T^n L(M, m_0; T^n N, (n_0, \bar{0}_n))_\omega \otimes T^n_{(n_0, \bar{0}_n)} N$, where $\bar{0}_n \in T^n N$ is the zero section for each $n \in \mathbf{N}$. Therefore, it is possible to vary values of differentials $D^j f$ for $j = 0, ..., n$ in the notation $T^n f := (f, Df, D^2 f, ..., D^n f)$ with $D^0 f := f$ for elements $f \in G$" both in the case of the diffeomorphism group and the loop group up to the corrections $o(d^{n+1}_{G'}(f, \psi))$. Then $D^n_h L_h(g)$ can be expressed through $D^j h$ and $D^j g$ with $1 \le j \le n$, where $h, g \in G$, hence it is possible to vary coefficients a^l_1, a^l_2 in the real case and a^l, b^l in the non-Archimedean case.

Take for example, the Gaussian measure on X in the real case induced from the Gaussian measure on the corresponding Banach space and given with the help of non-degenerate symmetric positive definite operator of trace

class. In the non-Archimedean case each Banach space over a local field $\mathbf{L}$ is isomorphic with $c_0(\alpha, \mathbf{L})$, where α is an ordinal and elements of $c_0(\alpha, \mathbf{L})$ have the form $x = (x_j : j \in \alpha, x_j \in \mathbf{L})$ such that $\|x\| := \sup_j |x_j| < \infty$ and for each $\epsilon > 0$ a set $\{j : |x_j| > \epsilon\}$ is finite [34]. For each separable manifold M we have $card(\alpha) \le \aleph_0$. In the latter case take, for example, the following non-Archimedean analog η of the Gaussian measure: each projection η_j of η on $\mathbf{L}e_j$ has a density $\eta_j(dx) = F_j exp(-|x|^2 s_j) v(dx)$, where $\sum_j s_j^{-1} < \infty$, $e_j := (0, ..., 0, 1, 0...)$ with 1 on the j-th place, v is the Haar measure on $\mathbf{L}$ with $v(B(\mathbf{L}, 0, 1)) = 1$ and constants $F_j > 0$ are chosen such that $\eta_j(\mathbf{L}) = 1$ (see also §2.9).

Let ψ_j^b be C^∞-curves in G' such that $(\partial\psi_j^b/\partial b)|_{b=0}$ are linearly independent vectors in $T_{g_k}G'$ and $\psi_j^b|_{b=0} = g_k$ and $R(\psi_j) \cap R(\psi_l) \cap (g_k W_k) = \{g_k\}$ for each $j \ne l$, $b \in \mathbf{L}$, where either $X = M$ or $X = G$, $j = 1, ..., n$, $n \in \mathbf{N}$, $R(\psi)$ denotes the range of ψ, that is, $R(\psi) := \{\psi^b : b \in \mathbf{L}\}$, $g_k \in G'$, S_k is open in $\tilde{\Gamma}_X$, $\zeta_k : S_k \to Q_k$ are local diffeomorphisms of open subsets S_k in $\tilde{\Gamma}_X$ and Γ_X respectively, $\gamma \in g_k S_k$ (see also §2.4).

There are embeddings $L^2(B_K^n, m_n, \mathbf{C}) \hookrightarrow L^2(B_X^n, m_n, \mathbf{C}) \hookrightarrow L^2(\Gamma_X, P_m, \mathbf{C}) \hookrightarrow L^2(\tilde{\Gamma}_X, P_m, \mathbf{C})$, where m_n denotes the restriction of P_m on B_X^n. For each $x \in X$ there exists $K \in \{K_l : l \in \mathbf{N}\}$ such that $x \in Int(K)$. Then Γ_K is the disjoint union of $\{B_K^n : n \in \mathbf{N_o}\}$. On the other hand, $P_{K,m}|_{Bf(K^n)} = m_{K,n}$ in accordance with §3.1, where $m_{K,n}$ is equivalent with m_K^n, where m_K denotes the restriction of m on $Bf(K)$ and m_K^n is the product of n copies of m_K. Then the condition $det(\Psi(\gamma)) = 0$ defines a submanifold $\tilde{\Gamma}_{X,\Psi}$ in $\tilde{\Gamma}_X$ of codimension over $\mathbf{L}$,

(i) $codim_{\mathbf{L}} \tilde{\Gamma}_{X,\Psi} \ge 1$, where $\Psi(\gamma)$ is a matrix dependent from $\gamma \in \tilde{\Gamma}_X$ with indices of rows and columns j and $l = 1, ..., n$ for $n \in \mathbf{N}$ with matrix elements $\Psi_{l,j}(\gamma) := D_{\phi_j}^{2l}(\rho(\phi_j, g))^{1/2}$ in the real case and with the corresponding pseudodifferentials by parameters $b_j \in B(\mathbf{L}, 0, 1)$ and for $\phi_j^{b_j}$ in the non-Archimedean case instead of differentials (see also [18, 28]). In the non-Archimedean case $\tilde{\Gamma}_X$ has the analytic atlas $At(\tilde{\Gamma}_X) = \{(V_j, \omega_j) : j \in \mathbf{N}\}$ with disjoint clopen charts, also G' has disjoint clopen charts and the analytic atlas $At(G') = \{(U_j, \phi_j) : j\}$, hence curves ψ_j^b can be chosen locally analytic with a restriction on the corresponding neighbourhood U_1 of e in G' being analytic, where $b \in \mathbf{L}$. Substitution of differentiation on pseudodifferentiation along ϕ_j^b by parameter $b \in B(\mathbf{L}, 0, 1)$ produces by formula $det(\Psi(\gamma)) = 0$ an analytic submanifold $\tilde{\Gamma}_{X,\Psi}$ in $\tilde{\Gamma}_X$ with $codim_{\mathbf{L}} \tilde{\Gamma}_{X,\Psi} \ge 1$.

Since for equivalent measures such regular representations are equivalent, we can consider infinitely differentiable or pseudodifferentiable measures in the real and non-Archimedean cases respectively. There is the following equality $\lim_{(m^n(B)\to 0,\infty>m^n(B)>0)} m^n(B)exp(-m^n(B))/((m^n)^\psi(B)exp(-(m^n)^\psi(B))) =: m^n(dx)exp(-m^n(dx))/((m^n)^\psi(dx)exp(-(m^n)^\psi(dx))) = \rho_{m^n}(\psi,x)$, since $\rho_{m^n}(\psi,x)$ is continuous on $G'\times\tilde{X}^n$ and $\lim_{(m^n(B)\to 0,\infty>m^n(B)>0,x\in B)} exp(-m^n(B)[1-\int_B \rho_{m^n}(\psi,y)m^n(dy)/m^n(B)]) = 1$ for balls B in $\tilde{X}^n$ such that $x\in Int(B)$, where $(m^n)^\psi(E) := m^n(\psi^{-1}E)$ for each Borel subset $E\in Bf(\tilde{X}^n)$, $\psi\in G'$, $x\in\tilde{X}^n$.

In the case $X = G$ each space $L^2(B^r_X, m_r, \mathbf{C})$ has the embedding into $L^2(\tilde{\Gamma}_X, P_m, \mathbf{C})$, where $m_r := P_m|_{B^r_X}$. It was supposed above that the quasi-invariance factor $\rho_m(\psi,x)$ of the quasi-invariant measure m on $Bf(X)$ relative to G' is continuous on $G'\times X$, consequently, $\rho_{m^r}(\psi,\eta)$ and $\rho_{m_r}(\psi,\gamma^r)$ and $\rho_{P_m}(\psi,\gamma)$ are continuous on $G'\times X^r$ and $G'\times B^r_X$ and $G'\times\Gamma_X$ respectively, where $\psi\in G'$, $\eta\in X^r$, $\gamma^r\in B^r_X$, $\gamma\in\Gamma_X$. Hence due to the definiton of P_m there is the equality: $lim_{r\ge n, r\to\infty}\rho_{m_r}(\psi,\gamma^n) = \rho_{P_m}(\psi,\gamma^n)$ for each $\psi\in G'$, $\gamma^n\in B^n_X$. If $f\in\bar{H} := L^2(\tilde{\Gamma}_X, P_m, \mathbf{C})$ is such that $(f(g), (\rho_{P_m}(\phi,g))^{1/2})_{\bar{H}} = 0$ for each $\phi\in G'\cap W$, then differentials of these scalar products by ϕ are zero. In view of the above embeddings and formula 2.4(i) and in view of condition (i) this means that $f=0$, since for each $n\in\mathbf{N}$ there are $\phi_j\in G'\cap W$ such that $det\Psi(\gamma)\ne 0$ P_m-almost everywhere on $\tilde{\Gamma}_X$, $\gamma\in\tilde{\Gamma}_X$. If $\|f\|_{\bar{H}}>0$, then $P_m(supp(f))>0$, consequently, $P_m(G'supp(f)) = 1$, since $G'U = \tilde{\Gamma}_X$ for each open U in $\tilde{\Gamma}_X$ and for each $\epsilon>0$ there exists an open U, $U\supset supp(f)$, such that $P_m(U\setminus supp(f))<\epsilon$.

This means that the linear span over $\mathbf{C}$: $sp_{\mathbf{C}}\{Ch_{g_kS_k}\phi(g) : \phi(g) = \rho^{1/2}_{P_m}(h,g); h\in G'\}$ is dense in $L^2(g_kS_k, P_m, \mathbf{C})$, since $U'_lK_l\subset Int(K_{l+1})$ for each $l\in\mathbf{N}$ (see §2.9). Therefore, $sp_{\mathbf{C}}\{\phi(g) : \phi(g) = \rho^{1/2}_{P_m}(h,g); h\in G'\}$ is dense in $L^2(\tilde{\Gamma}_X, P_m, \mathbf{C})$ and a vector f_0 is cyclic for U_m, where $f_0(\gamma) = 1$ for each $\gamma\in\tilde{\Gamma}_X$. Then A_G" contains subalgebra of all operators of multiplication on functions from $L^\infty(\tilde{\Gamma}_X, P_m, \mathbf{C})$ and the remainder of the proof of theorem 3.9 is quite analogous with the proof of theorem 3.8 (certainly A_G" $\ne L^\infty(\tilde{\Gamma}_X, P_m, \mathbf{C})I$ for $G' = Diff^t(M)$, since the regular representation $U_m(h)$ of G' contains a family of cardinality $\mathsf{c} := card(\mathbf{R})$ of non-commuting operators from the set $\{U_m(h) : h\in G'\}$).

3.10. Theorem. (α). *If there exists a bounded operator* $T : L^2(\tilde{\Gamma}_X, P_m, \mathbf{C})\otimes H^q \to L^2(\tilde{\Gamma}_X, P_{m'}, \mathbf{C})\otimes H^{q'}$ *($H^q = \{0\}$ and P_m is from theorem 3.9 for*

$X = G$ or infinite-dimensional $X = M$ over the corresponding field $\mathbf{L}$ such that $L^2 \otimes \{0\} := L^2$) satisfying conditions (a, b):
(a) $TU^q_m(\psi) = U^{q'}_{m'}T$ for all $\psi \in G'$,
(b) there exists $\phi \in H^q$ such that $T(1 \otimes \phi) \ne 0$,
then P_m and $P_{m'}$ are equivalent.

(β). If there exists a bounded operator $V : L^2(G, \mu, \mathbf{C}) \to L^2(G, \mu', \mathbf{C})$ such that $VT^\mu(\psi) = T^{\mu'}(\psi)V$ for each $\psi \in G'$, where μ is a quasi-invaraint measure on G relative to G' and T^μ is the associated regular representation of G' from theorem 3.8, then μ and μ' are equivalent.

The proof is divided into several parts. At first the case (α) of $X = M$ finite-dimensional over the corresponding filed $\mathbf{L}$ is considered in subparagraphs **I-III**. The cases (α) of $X = G$ and infinite-dimensional $X = M$ over $\mathbf{L}$ and the cases (β) are considered in §3.10.**IV**.

I. Suppose that $\|\phi\| = 1$ and T is a contraction operator. Take $X_n := K_n$, where $n \in \mathbf{N}$ and K_n are the same as in §2.9. In the case $X = M$ we put $Y = X_n$, $\mu = P_m$, $\mu' = P_{m'}$, μ_1 and μ_2 are equal to the image measure of μ in accordance with the maps: $\gamma \mapsto (\gamma \cap Y) =: \gamma_1$, $\gamma \mapsto (\gamma \cap Y^c) =: \gamma_2$. Apart from the case $X = M$, for $X = G$ we suppose that $Y = X$, since G' acts on G transitively and $supp(L_\psi) := cl\{g \in G : \psi g \ne g\} = G$ for each $\psi \ne e$, because G' is a dense subgroup of G and from $hg = g$ it follows $h = e$, where $h, g \in G$. In the case of $Diff^t(X)$ there exists a bounded operator $T_Y : L^2(\Gamma_Y, \mu_1, \mathbf{C}) \otimes H^q \to L^2(\Gamma_Y, \mu'_1, \mathbf{C}) \otimes H^{q'}$ such that

(i) $T_Y F(\gamma, a') = \int_{\Gamma_{Y^c}} TF(\gamma_1, \gamma_2, a')\mu'_2(d\gamma_2)$. Then $L^2(\Gamma_Y, \mu_1, \mathbf{C})$ is embeddable as a closed subspace into $L^2(\Gamma_X, \mu, \mathbf{C})$ by the map $L^2(\Gamma_Y, \mu_1, \mathbf{C}) \ni f(\gamma) \mapsto \hat{f}(\gamma) := f(\gamma \cap Y) \in L^2(\Gamma_X, \mu, \mathbf{C})$. Therefore, $T_Y F$ depends on (γ_1, a') and $T_Y F(\gamma, a'_\sigma) = q'(\sigma)^{-1} T_Y F(\gamma, a')$ for all $\sigma \in \Sigma_{n'}$, where $a'_\sigma = (i_{\sigma(1)}, ..., i_{\sigma(n')})$ for each $a' = (i_1, ..., i_{n'}) \in \tilde{\mathbf{N}}^{n'}$. Then for $Diff^t(X)$: $\|T_Y F\|^2 = \sum_{a' \in \tilde{\mathbf{N}}^{n'}} \int_{\Gamma_Y} \|T_Y F(\gamma_1, a')\|^2_{W'} \mu'_1(d\gamma_1) \le \sum_{a' \in \tilde{\mathbf{N}}^{n'}} \int_{\Gamma_X} \|T_Y F(\gamma, a')\|^2_{W'} \mu'_1(d\gamma)$ $\le \int_{\Gamma_Y} \int_{\Gamma^c_Y} \sum_{a' \in \tilde{\mathbf{N}}^{n'}} \|TF(\gamma_1, \gamma_2, a')\|^2_{W'} \mu'_1(d\gamma_1)\mu'_2(d\gamma_2) = \|TF\|^2 \le \|F\|^2$ in the case $X = M$, consequently, $\|T_Y\| \le 1$ and T_Y is a contraction too. When $\psi \in Diff^t(Y)$, then $\sigma(\psi, \gamma)$ is independent of γ_2. Hence $T_Y U^q_m(\psi) = U^{q'}_{m'}(\psi) T_Y$ for each $\psi \in Diff^t(Y)$.

There exists the decomposition of Γ_X into disjoint union of subsets $B^r_{X_k} \times \Gamma_{X \setminus X_k}$ for $r = 0, 1, 2, ...$, where each such subset is invariant relative to $Diff^t(X_k)$, where k is fixed and $B^r_{X_k}$ is the set of r-point subsets in X_k, $B^0_{X_k} := \{\emptyset\}$ is the singleton. Therefore, $L^2(\Gamma_X, \mu, \mathbf{C}) \otimes H = \bigoplus_{r=0}^\infty (L^2(B^r_{X_k} \times$

$\Gamma_{X\setminus X_k}, \mu_r, \mathbf{C}) \otimes H$, where μ_k is a restriction of μ on $B^r_{X_k} \times \Gamma_{X\setminus X_k}$ (see also §2.13). Hence $L^2(B^r_{X_k} \times \Gamma_{X\setminus X_k}, \mu_r, \mathbf{C}) \otimes H = \bigoplus_i (L^2(B^r_{X_k} \times \Gamma_{X\setminus X_k}, \mu_r, \mathbf{C}) \otimes W^i_r \otimes C^i_r)$, where W^i_r are spaces in which irreducible pairwise non-equivalent unitary representations q^i_r of the symmetric group Σ_r act, C^i_r denote spaces in which Σ_r acts trivially. Each term in the direct sum is invariant under $Diff^t(X_k)$ such that from $\psi \in Diff^t(X_k)$ and $\gamma \in B^r_{X_k} \times \Gamma_{X\setminus X_k}$ it follows that $\sigma(\psi, \gamma) \in \Sigma_r$.

In view of lemma 2.14 the measure μ_r is equivalent with $\mu'_r \times \mu"_r$. Hence there exists the isomorphism $\tau_r : L^2(B^r_{X_k} \times \Gamma_{X\setminus X_k}, \mu_r, \mathbf{C}) \to L^2(B^r_{X_k}, \mu'_r, \mathbf{C}) \otimes L^2(\Gamma_{X\setminus X_k}, \mu"_r, \mathbf{C})$ given by the following formula: $\tau_r F := [\mu_r(d\gamma)/(\mu'_r(d\gamma')\mu"_r(d\gamma"))]^{1/2} F(\gamma)$. Hence there exists isomorphism $\tau_r : L^2(B^r_{X_k} \times \Gamma_{X\setminus X_k}, \mu_r, \mathbf{C}) \otimes W^i_r \otimes C^i_r \to L^2(B^r_{X_k}, \mu'_r, \mathbf{C}) \otimes W^i_r \otimes L^2(\Gamma_{X\setminus X_k}, \mu"_r, \mathbf{C}) \otimes C^i_r$. Therefore, for finite-dimensional manifolds M over $\mathbf{R}$ or the local field $\mathbf{L}$ considered here for $\mu = P_m$ there is true the following lemma (for finite-dimensional real M see also lemma 3.2 [38]).

3.11. Lemma. *Under the isomorphism τ_r the operator $U(\psi) := U^q_m(\psi)$ for each $\psi \in Diff^t(X_k)$ transforms into $\tau_r U(\psi) \tau_r^{-1} = U^i_r(\psi) \otimes I$, where I is the unit operator in $L^2(\Gamma_{X\setminus X_k}, \mu"_r, \mathbf{C}) \otimes C^i_r$ and U^i_r is the operator in the space $L^2(B^r_{X_k}, \mu'_r, W^i_r)$ such that $(U^i_r(\psi)F)(\gamma^r) = \rho^{1/2}_{\mu'_r}(\psi, \gamma^r)\, q^i_r(\sigma_r(\psi, \gamma^r))F(\psi^{-1}\gamma^r)$, where $\gamma^r \in B^r_{X_k}$.*

II. A unitary representation $Q : \Sigma_\infty \to U(H^q)$ such that $Q(\sigma) : \phi(a) \mapsto \phi(\sigma^{-1}a)$ restricted on Σ_r splits into the direct sum of subspaces: $H^q = \bigoplus_i W^i_r \otimes C^i_r$, that is, $Q(\sigma)\phi = \sum_i \{q^i_r \otimes id\}\phi_{r,i}$, $\phi = \sum_i \phi_{r,i}$, where $\phi_{r,i} \in W^i_r \otimes C^i_r$, q^i_r are the ireducible and pairwise distinct representations of Σ_r, $\Sigma_\infty := ind - \lim_r \Sigma_r$. Since $\Gamma_Y = \bigcup_{r=0}^\infty B^r_Y$ is the disjoint union of B^r_Y, then there are the following orthogonal decompositions: $L^2(\Gamma_Y, \mu_1, \mathbf{C}) = \bigoplus_{r=0}^\infty L^2(B^r_Y, \mu_1, \mathbf{C})$ and $L^2(\Gamma_Y, \mu_1, \mathbf{C}) \otimes H^q = \bigoplus_{r,i} \phi_\mu(r,i)$, where $\phi_\mu(r,i) := L^2(B^r_Y, \mu_1, \mathbf{C}) \otimes W^i_r \otimes C^i_r$ are invariant subspaces of the representation $U^q_m|_{Diff^t(Y)}$. Therefore, $U^q_m(\psi) = U^{r,i}_\mu(\psi) \otimes id$ on $\phi_\mu(r,i)$, where

$(II.i)$ $U^{r,i}_\mu(\psi)(F \otimes w^i_r) = \rho^{1/2}_{\mu_1}(\psi, \gamma_1) F(\psi^{-1}(\gamma_1)) q^i_r(\sigma(\psi,\gamma)) w^i_r$ for $F \in L^2(B^r_Y, \mu_1, \mathbf{C})$ and $w^i_r \in W^i_r$. The irreducible unitary representations $U^{r,i}_\mu$ and $U^{r',i'}_\mu$ are equivalent if and only if $i = i'$ and $r = r'$.

Hence there exists the unique integer J_i such that either $T_Y\phi_\mu(r,i) = 0$ or $T_Y\phi_\mu(r,i) \subset \phi_{\mu'}(r, J_i)$ and the representations q^i_r and ${q'}^{J_i}_{r'}$ are equivalent, consequently, $J_i \ne J_k$ for each $i \ne k$.

There exist intertwining operators $\omega_{r,i} : W^i_r \to {W'}^{J_i}_r$ of the representa-

tions q_r^i and ${q'}_r^{J_i}$. We denote by J_Y the unitary operator $J_Y : L^2(B_Y^r, \mu_1, \mathbf{C}) \to L^2(B_Y^r, {\mu'}_1, \mathbf{C})$ given by the following formula: $J_Y F(\gamma_1) := (\mu_1(d\gamma_1)/{\mu'}_1(d\gamma_1))^{1/2} F(\gamma_1)$. Hence

$(II.ii)$ $U_{\mu'}^{r,J_i}(\psi) T_{r,i} = T_{r,i} U_{\mu}^{r,i}(\psi)$ for each $\psi \in Diff^t(Y)$, where $T_{r,i} = J_Y \otimes \omega_{r,i} : L^2(B_Y^r, \mu_1, \mathbf{C}) \otimes W_r^i \to L^2(B_Y^r, {\mu'}_1, \mathbf{C}) \otimes {W'}_r^{J_i}$.

III. Using the general fact of the representation theory of topological groups from §III of the proof of theorem 3.1 in [36] we get for each (r, i): either exists a bounded operator $U_{r,i} : C_r^i \to {C'}_r^{J_i}$ such that $T_Y|_{\phi_\mu(r,i)} = T_{r,i} \otimes U_{r,i}$ or $T_Y \phi_\mu(r, i) = 0$. Hence for $Diff^t(Y)$ there is the following equality:

$(III.i)$ $T_Y(1 \otimes \phi)(\gamma, a') = \sum'_{r,i} T_{r,i} \otimes U_{r,i}(\chi_{B_Y^r} \otimes \phi_{r,i})(\gamma, a') = (mu_1(d\gamma_1)/{\mu'}_1(d\gamma_1) \sum'_{r,i} \chi_{B_Y^r}(\gamma_1)(\omega_{r,i} \otimes U_{r,i})(\phi_{r,i})(a')$, where $\sum'$ is a sum for (r, i) such that $T_Y \phi_\mu(r, i) \ne 0$, $\phi = sum_i \phi_{r,i}$, $\phi_{r,i} \in W_r^i \otimes C_r^i$, χ_A is the characteristic function of the subset A. Then $\|\sum'_{r,i} \chi_{B_Y^r}(\gamma_1)(\omega_{r,i} \otimes U_{r,i})(\phi_{r,i})(a')\|^2_{W'} \le \sum_r \chi_{B_Y^r}(\gamma_1) \sum_i \|\phi_{r,i}\|^2 = 1$.

In the case of $Diff^t(X)$ if P_m and $P_{m'}$ are mutually singular, then $\lim_{k\to\infty} T_{X_k}(1 \otimes \phi)(\gamma, a')$ converges to $T(1 \otimes \phi)(\gamma, a')$ for $P_{m'}$-a.e. γ due to the martingale convergence theorem, but $\lim_{k\to\infty}(\mu_1(d\gamma_1)/{\mu'}_1(d\gamma_1))^{1/2}(\gamma_1) \sum'_{r,i} \chi_{B_Y^r}(\gamma_1)(\omega_{r,i} \otimes U_{r,i})(\phi_{r,i})(a') = 0$ for P_m-a.e. γ, hence $T(1 \otimes \phi) = 0$, which contradicts the assumption of this theorem.

IV. In view of theorem 3.9 representations U_m are irreducible for $X = G$ or infinite-dimensional $X = M$ over the field $\mathbf{L}$. It was proved in §3.9 that

$(IV.i)$ the weak closure of subalgebra generated by the family $\{U_m(h) : h \in G'\}$ in the algebra of bounded linear operators $L(\bar{H})$ contains all operators of multiplication on functions from the space $L^\infty(\tilde{\Gamma}_X, P_m, \mathbf{C})$, where $\bar{H} := L^2(\tilde{\Gamma}_X, P_m, \mathbf{C})$. If measures P_m and $P_{m'}$ are singular, then

$(IV.ii)$ either $\sup_{(\gamma \in \tilde{\Gamma}_X)} |P_{m'}(d\gamma)/P_m(d\gamma)| = \infty$ or $\sup_{(\gamma \in \tilde{\Gamma}_X)} |P_m(d\gamma)/P_{m'}(d\gamma)| = \infty$, where $P_{m'}(d\gamma)/P_m(d\gamma) := \lim_{(P_m(B)\to 0, \infty > P_m(B) > 0, \gamma \in B)} P_{m'}(B)/P_m(B) \in [0, \infty]$, $[0, \infty] := ([0, \infty) \cup \{\infty\})$, $[0, \infty) := \{x : x \in \mathbf{R}, 0 \le x\}$, $B \in Bf(\tilde{\Gamma}_X)$. In view of the existence of the intertwining operator T of U_m with $U_{m'}$ there exists an isomorphism of Hilbert spaces $\tau : L^2(\tilde{\Gamma}_X, P_m, \mathbf{C}) \to L^2(\tilde{\Gamma}_X, P_{m'}, \mathbf{C})$, which has a continuous extension to an isomorphism of Banach spaces $\tau : L^\infty(\tilde{\Gamma}_X, P_m, \mathbf{C}) \to L^\infty(\tilde{\Gamma}_X, P_{m'}, \mathbf{C})$ due to condition $(IV.i)$. On the other hand, in view of condition $(IV.ii)$ there exists a sequence $f_n \in L^2(\tilde{\Gamma}_X, P_m, \mathbf{C})$ such that $C_1 a_n \le b_n \le C_2 a_n$ for each $n \in \mathbf{N}$ and $\lim_{n\to\infty} c_n < \infty$ and $\lim_{n\to\infty} d_n = \infty$, where C_1 and C_2 are positive constants, $a_n := \|f_n\|_{L^2(\tilde{\Gamma}_X, P_m, \mathbf{C})}$, $b_n := \|\tau f_n\|_{L^2(\tilde{\Gamma}_X, P_{m'}, \mathbf{C})}$, $c_n := \|f_n\|_{L^\infty(\tilde{\Gamma}_X, P_m, \mathbf{C})}$, $d_n := \|\tau f_n\|_{L^\infty(\tilde{\Gamma}_X, P_{m'}, \mathbf{C})}$, since there are sequences $\{y_n : 0 < y_n < \infty, n \in \mathbf{N}\}$

such that $\sum_n y_n^{-2} < \infty$, but $\sum_n y_n^{-1} = \infty$. This means that singularity of P_m with $P_{m'}$ leads to the contradiction, consequently, P_m and $P_{m'}$ are equivalent. The cases (β) are proved analogously with μ instead of P_m and G instead of $\tilde{\Gamma}_X$ due to theorem 3.8.

3.12. Corollary. (α). *If U^q_m and $U^{q'}_{m'}$ are equivalent as unitary representations, then P_m and $P_{m'}$ are equivalent as measures.*

(β). *If T^μ and $T^{\mu'}$ are equivalent as unitary representations, then μ and μ' are equivalent as measures.*

3.13. Theorem. (α). *If P_m and $P_{m'}$ are equivalent, $n = n'$, unitary representations q and q' of Σ_n and $\Sigma_{n'}$ are equivalent (in the case $Diff^t(M)$ of the real manifold M with the additional condition $dim_{\mathbf{R}} M > 1$; $H^q = \{0\}$ and $q = I$ for $X = G$ or for infinite-dimensional manifold $X = M$ over the field $\mathbf{L}$). Then the unitary representations U^q_m and $U^{q'}_{m'}$ are equivalent.*

(β). *If μ and μ' from theorem 3.8 are equivalent quasi-invaraint measures on G relative to G', then the regular unitary representations T^μ and $T^{\mu'}$ are equivalent.*

Proof. The cases (α) for $X = M$ infinite-dimensional over the field $\mathbf{L}$ or $X = G$ and (β) follow from the fact that $\tau : L^2(Z, \mu, \mathbf{C}) \to L^2(Z, \mu', \mathbf{C})$ given by the following formula $(\tau f)(x) = (\mu(dx)/\mu'(dx))^{1/2} f(x)$ is the linear topological isomorphism and the intertwining operator of two regular representations in these Hilbert spaces, where either $Z = \tilde{\Gamma}_X$ with $\mu = P_m$ or $Z = G$ with a quasi-invariant measure μ relative to G' respectively.

It remains only to consider the case of the non-Archimedean manifold $X = M$ with $dim_{\mathbf{L}} M < \infty$, since the case of real M was proved in §4 [38]. The measure m on M is supposed to be the restriction of the Haar measure from $\mathbf{L^n}$ on M (see §2.9). Let $Diff^t(X, m)$ be a subgroup of $G' = Diff^t(X)$ consisting of diffeomorphisms ψ, for which $\rho_m(\psi, x) = 1$ for each $x \in M$, where $1 \le t \le \infty$. In the case of $X = M = \mathbf{R}$ we have $Diff^t(X, m) = \{e\}$, but in the non-Archimedean case each $\psi \in G'$ with $sup_{x \in M} |\psi'(x) - I| < 1$ belongs to $Diff^t(X, m)$. For example, if a countable family of disjoint balls $B_j := B(\mathbf{L^n}, x_j, r)$ with $j \in \Upsilon \subset \mathbf{N}$ of radius $0 < r < \infty$ is contained in M and if $\psi \in G'$ is such that $\psi(B_j) = B_{\zeta(j)}$ for each $j \in \Upsilon$, $\psi|_{B_j}(x_j + z) = x_{\zeta(j)} + \phi_j(z)$ for each $z \in B(\mathbf{L^n}, 0, r)$, $\psi|_{(M \setminus \bigcup_j B_j)} = id$, $\phi_j : B(\mathbf{L^n}, 0, r) \to B(\mathbf{L^n}, 0, r)$ are diffeomorphisms with $\sup_{(x \in B(\mathbf{L^n}, 0, r))} |\phi'_j(x) - I| < 1$ for each $j \in \mathbf{N}$, where $\zeta : \Upsilon \to \Upsilon$ is a bijection, then $\psi \in Diff^t(M, m)$, since $B(\mathbf{L^n}, 0, r)$ are clopen in $\mathbf{L^n}$ and the valuation group $\{|x|_{\mathbf{L}} : 0 \ne x \in \mathbf{L}\}$

is discrete in $(0,\infty)$. Therefore, in the non-Archimedean case there is not any restriction on $dim_{\mathbf{L}}M$ from below. If $A \subset \Gamma_X$ is invariant by $mod(P_m)$ subset of Γ_X and $P_m(A) > 0$, then $P_m(A) = 1$. Indeed, if $P_m(\Gamma_X \setminus A) > 0$, then there exists $\psi \in Diff^t(X,m)$ such that $P_m((\Gamma_X \setminus A) \cap \psi(A)) > 0$, since m is quasi-invariant relative to G' with the continuous quasi-invariant factor $\rho_m(h,x)$ by $(h,x) \in G' \times \Gamma_X$ and $P_m((hA) \cap B)$ is the continuous function by $h \in G'$ for each A and $B \in Af(\Gamma_X, P_m)$, where $Af(\Gamma_X, P_m)$ denotes the completion of the Borel σ-field $Bf(\Gamma_X)$ by the ergodic measure P_m. In view of the invariance of A we have $P_m((\Gamma_X \setminus A) \cap A) > 0$, which is a contradiction, hence $P_m(\Gamma_X \setminus A) = 0$.

The restriction of the regular unitary representation $U_m|_{Diff^t(X,m)}$ is given by the following formula:

$(U_m(\psi)f)(\gamma) = f(\psi^{-1}\gamma)$ for each $\psi \in Diff^t(X,m)$. Then $f_0(\gamma) = 1$ for each $\gamma \in \Gamma_X$ is the unique vector in $L^2(\Gamma_X, P_m, \mathbf{C})$ such that $\mathbf{C}f_0$ is invariant relative to $U_m(\psi)$ for each $\psi \in Diff^t(M,m)$. The Poisson measure P_m can be considered with a parameter $\lambda > 0$, that is with λm instead of m. Let $u_m(\psi)$ be a spherical function given by the following formula: $u_m(\psi) = (U_m(\psi)f_0, f_0)$, where $(*,*)$ denotes the scalar product in $L^2(\Gamma_X, P_m, \mathbf{C})$. Then $u_m(\psi) = exp(\int_X (\rho_m^{1/2}(\psi,x) - 1)m(dx))$, since $u_m(\psi) = \int_{\Gamma_X} (\prod_{x\in\gamma} \rho^{1/2}(\psi,x)) P_m(d\gamma)$ and for $supp(\psi) \subset Y$ with $m(Y) < \infty$ we have $u_m(\psi) = \sum_{n=0}^{\infty} \int_{B_Y^n} (\prod_{x\in\gamma} \rho^{1/2}(\psi,x)) P_m|_{B_Y^n}(d\gamma)$. Therefore, we get the following theorem.

3.14. Theorem. (α). *The representations $U_{\lambda_1 m}$ and $U_{\lambda_2 m}$ of $Diff^t(M)$ (with the restriction $dim_{\mathbf{R}}M > 1$ for M over $\mathbf{R}$ and $0 < dim_{\mathbf{L}}M$ in the non-Archimedean case) for $\lambda_1 \ne \lambda_2$ are inequivalent.*

(β). *The representations $U_{\lambda_1 m}$ and $U_{\lambda_2 m}$ of G' in $L^2(\tilde{\Gamma}_G, P_{\lambda_j m}, \mathbf{C})$ with $j = 1,2$ respectively are inequivalent for $\lambda_1 \ne \lambda_2$.*

Proof. (α). For $dim_{\mathbf{L}}M < \infty$ this follows from the fact $u_{\lambda_1 m} \ne u_{\lambda_2 m}$.

(β). In view of the Kakutani theorem [5] two Poisson measures $P_{\lambda_1 m}$ and $P_{\lambda_2 m}$ are singular, hence by theorem 3.10 representations $U_{\lambda_1 m}$ and $U_{\lambda_2 m}$ are inequivalent.

3.15. Note. By the given above representations it is possible to produce new with the help of the following construction. Let G' be a group acting from the left on a C^∞-manifold Y (or analytic with disjoint charts in the non-Archimedean case over the local field $\mathbf{L}$) such that on Y is given a σ-additive σ-finite quasi-invariant non-negative measure μ with a continuous

quasi-invariant factor $\rho_\mu(\psi, y)$ by $(\psi, y) \in G' \times Y$. In the real case let us consider a space $\mathsf{F}(Y)$ of generalised functions on Y. For example, if there is a unitary regular representation T of G' in $L^2(Y, \mu, \mathbf{C})$, then $\mathsf{F}(Y)$ is a space of continuous linear functionals on $L^2(Y, \mu, \mathbf{C})$, hence $\mathsf{F}(Y)$ is isomorphic with $L^2(Y, \mu, \mathbf{C})$. Let ν be a measure on $\mathsf{F}(Y)$ given with the help of its characteristic function $\int_{\mathsf{F}(Y)} exp(i < F, f >)\nu(dF) = exp(-\|f\|^2/2)$, where $\|*\|$ is a norm in $L^2(Y, \mu, \mathbf{C})$. Such ν is called the standard Gaussian measure in $\mathsf{F}(Y)$. Then a new representation $\tilde{U} := EXP_\beta T$ is given by the following formula:

$(\tilde{U}(\psi)\Phi)(F) := exp(i < F, \beta(\psi) >)\Phi(T^*(\psi)F)$, where $< T^*(\psi)F, f >=< F, T(\psi)f >$, $< F, f >$ is a value of a functional $F \in \mathsf{F}(Y)$ on a function $f \in L^2(Y, \mu, \mathbf{C})$, β is a 1-cocycle such that $[\beta(\psi)](y) := \rho_\mu^{1/2}(\psi, y) - 1$, $T(\psi) := \rho^{1/2}(\psi, y)f(\psi^{-1}y)$ for each $\psi \in G'$, $\Phi \in L^2(\mathsf{F}, \nu, \mathbf{C})$, $T^*(\psi) = T^{-1}(\psi) = T(\psi^{-1})$. If to substitute β on $s\beta$, where $s \in \mathbf{R}$, then it produces the one-parameter family $\tilde{U}_s := EXP_{s\beta}T$. There is an equality $\lim_{\psi \to e} \beta(\psi, y) = 0$ for each $y \in Y$. When $Y = G$ or $Y = \tilde{\Gamma}_X$ and μ is as in theorems 3.8 or 3.9, then for $s \ne 0$ representation $\tilde{U}_s$ is irreducible as follows from the proof of theorems 3.8, 3.9, since the linear span of non-linear functionals $exp(i < F, \beta(\psi) >)$ is dense in $L^2(\mathsf{F}(Y), \nu, \mathbf{C})$ (see also §4 in [38]). For $X = M$ with $dim_{\mathbf{L}} M < \infty$ and $s \ne 0$ the representations $\tilde{U}_s$ and $U_{s^2\mu}$ in $L^2(\Gamma_M, P_\mu, \mathbf{C})$ are equivalent, since $(\tilde{U}_s\psi\Phi_0, \Phi_0) = exp(\int_M(\rho^{1/2}(\psi, x) - 1)\mu(dx)) = u_{s^2}(\psi)$, where $\Phi_0(F) = 1$ for each $F \in \mathsf{F}(\Gamma_M)$, $\Phi_0 \in L^2(\mathsf{F}(\Gamma_M), \nu, \mathbf{C})$.

3.16. Note. It follows from [5, 28, 37], that on Z there are infinite families of orthogonal measures, restrictions of which on U are quasi-invariant relative to G' and have continuous quasi-invariance factor on $G' \times X$, where either $X = M$ or $X = G$ respectively, $W'U \subset V$, W' is open in G' and U is open in Z. Due to the general procedures of construction of measures on X outlined above on infinite-dimensional M over the corresponding field or on G there are infinite families of orthogonal and as well singular measures, since measures on these infinite-dimensional manifolds M and G are induced from the corresponding Banach spaces Z due to the local diffeomorphisms $A : W \to V$, where W is open in M or G and V is open in Z. Therefore, the last two theorems show that there exists an infinite family of non-equivalent unitary representations of G' for $X = G$ and also for $G' = Diff^t(M)$ for the infinite-dimensional manifold M over the corresponding field, since in these cases on G and M there exist infinite families of orhtogonal measures. The

unitary group $U(l_2)$ of the standard Hilbert space l_2 over $\mathbf{C}$ has the topological density c, when $U(l_2)$ is in its standard topolgy induced by the operator norm in the space of linear bounded operators $L(l_2)$ on l_2, since $l_2^{\mathbf{N}}$ in the box topology has a density $\aleph_0^{\aleph_0} = \mathsf{c}$. When $U(l_2)$ is considered as a topological space in its strong topology [9], then its topological density is $\aleph_0$, since $l_2^{\mathbf{N}}$ in the product Tychonoff topology has density $\aleph_0$ [7]. Therefore, the cardinality of distinct unitary strongly continuous representations $T : G' \to U(l_2)$ for topological group with density $\aleph_0$ do not exceed c, since $\mathsf{c}^{\aleph_0} = \mathsf{c}$ [7]. This is important difference of the theory of such non-locally compact topological groups with the theory of compact groups. In the latter case all irreducible unitary representations arise as irreducible components of the regular representation associated with the Haar measure, but for the considered here cases of groups this is not true, since there are infinite families of non-equivalent unitary representations on such groups. There are considered M and G and $\tilde{\Gamma}_X$ with countable bases of topology and real-valued measures. The family Ψ of distinct σ-additive Borel measures on these spaces have the cardinality $card(\Psi) = card(\mathbf{R}^{\mathbf{N}}) = card(\mathbf{R}) =: \mathsf{c}$. In view of theorems 3.10, 3.13, 3.14 and the criteria of orthogonality and singularity of measures on infinite-dimensional spaces (using weak distributions, product measures, Kakutani theorem and its non-Archimedean analog [5, 28] and the construction of measures on G or $\tilde{\Gamma}_X$ with the help of local diffeomorphisms of open subsets in these spaces and neigbourhoods of zero in the corresponding Banach spaces as in §2.9) there exist families Ψ_s of singular and Ψ_o of orthogonal measures such that $\mathsf{c} \le card(\Psi_o) \le card(\Psi_s) \le card(\Psi) = \mathsf{c}$, hence there are c non-equivalent unitary representations U_m of G' in $L^2(\tilde{\Gamma}_X, P_m, \mathbf{C})$ and also c inequivalent unitary representations of G' in $L^2(G, \mu, \mathbf{C})$, which were considered above, since $card(\Psi_s) = \mathsf{c}$.

3.17. Theorem. *There exist Abelian non-locally compact Banach-Lie groups G with quasi-invariant measures μ on G relative to dense subgroups G' such that the associated regular unitary representations T^μ of G' are irreducible, each one-parameter subgroup S of G is compact and a projection of μ on each one-parameter subgroup S is equivalent with the Haar measure on S.*

Proof. Let $l_{2,b}$ be a Hilbert space over $\mathbf{R}$ of elements $x = (x_j : j \in \mathbf{N}, x_j \in \mathbf{R})$ such that $\|x\|_{l_{2,b}}^2 := \sum_j |x_j j^b|^2 < \infty$. In particular $l_2 = l_{2,0}$. These spaces have standard orthonormal bases $e_k := (0, ..., 0, 1, 0, ...)$ with

1 on the k-th place, $k \in \mathbf{N}$. For a local field $\mathbf{L}$ let $c_{0,b}$ be a Banach space of elements $x = (x_j : j \in \mathbf{N}, x_j \in \mathbf{L})$ such that $\|x\| := \max_j |x_j|_{\mathbf{L}} p^{jb} < \infty$ and $\lim_{j\to\infty} |x_j| p^{jb} = 0$, where p is a prime number such that $\mathbf{L}$ is a finite algebraic extension of the field of p-adic numbers $\mathbf{Q_p}$. In particular $c_{0,0} = c_0$. If $b > 1$ (with $p^{-b} \in \Gamma'_{\mathbf{L}}$ in the non-Archimedean case) then the embeddings $J_b : l_{2,b} \hookrightarrow l_2$ and $S_b : c_{0,b} \hookrightarrow c_0$ are of trace class: $J_b e_k = a_k e_k$ with $a_k = k^{-b} \in \mathbf{R}$ and $\sum_k |a_k| < \infty$, $S_b e_k = v_k e_k$ with $v_k \in \mathbf{L}$ and $\sum_k |v_k| < \infty$, where $|v_k| = p^{-jb}$. On l_2 and c_0 there exist a Gaussian measure λ and a non-Archimedean analog η of a Gaussian measure quasi-invariant relative to $l_{2,b}$ and $c_{0,b}$ respectively such that their projections λ_k and η_k on one-dimensional subspaces $\mathbf{R}e_k$ and $\mathbf{L}e_k$ are the following: $\lambda_k(dx_k) = C_k exp(-x_k^2 s_k^2) w(dx_k)$ and $\eta_k(dy_k) = F_k exp(-|y_k|^2 p^{2k}) v(dy_k)$, where w and v are the Lebesgue and the Haar measures on $\mathbf{R}$ and $\mathbf{L}$ respectively such that $w([0,1]) = 1$, $v(B(\mathbf{L}, 0, 1)) = 1$, $s_k = k^{b'}$ for each $k \in \mathbf{N}$ with $1 < b' < b$, $C_k > 0$, $F_k > 0$, $\lambda_k(\mathbf{R}) = 1$, $\eta_k(\mathbf{L}) = 1$ (see [5, 28]). Consider an additive discrete subgroup E of $l_{2,b}$ consisting of elements $x \in l_{2,b}$ such that $x_j = n_j e_j$ for each $j \in \mathbf{N}$, where $n_j \in \mathbf{Z}$. Then $l_{2,b}/E =: H_b$ and $l_2/E =: H$ are the additive groups. The measures λ and η induce measures μ on H and ν on $B(c_0, 0, 1) := \{x \in c_0 : \|x\| \le 1\} =: B$. Then μ is quasi-invariant relative to H_b and ν is quasi-invariant relative to $B(c_{0,b}, 0, 1)$ with continuous quasi-invariance factors such that $\mu(H) = 1$ and $\nu(B) > 0$.

Let $L_n := sp_{\mathbf{R}}(e_1, ..., e_n)$ and $E_n := L_n \cap E$, so the latter is a discrete subgroup of L_n and $L_n/E_n =: V_n$ is a closed subgroup of H_b. Hence a projection $\pi_n : l_{2,b} \to L_n$, which has a continuous extension $\pi_n : l_2 \to L_n$ induces a quotient mapping $\bar{\pi}_n : H_b \to V_n$ with a continuous extension $\bar{\pi}_n : H \to V_n$ for each $n \in \mathbf{N}$. Therefore, the measure μ on H induces a measure μ_n on V_n such that $\mu_n(A) := \mu(\bar{\pi}_n^{-1}(A))$ for each $A \in Bf(V_n)$. In view of the equality $\lim_{n\to\infty} \rho_{\mu_n}(\bar{\pi}_n(\psi), \bar{\pi}_n(x)) = \rho_\mu(\psi, x)$ for each $\psi \in H_b$ and $x \in H$ it follows that $\rho_\mu(\psi, x) = \lim_{n\to\infty} (\sum_{z\in E_n} exp\{\sum_{l=1}^n [2(\psi + z, e_l)(x, e_l) - (\psi + z, e_l)^2] s_l^2\})$ $(\sum_{z\in E_n} exp\{\sum_{l=1}^n [2(z, e_l)(x, e_l) - (z, e_l)^2] s_l^2\})^{-1}$, since $(\pi_n(x), e_l) = (x, e_l)$ for each $x \in L_n$ with $n \ge l$. The Hilbert space $L^2(H, \mu, \mathbf{C})$ is isomorphic with a subspace $\{f : f \in L^2(l_2, \lambda, \mathbf{C}); f(x + z) = f(x)\ \lambda\text{-a.e. for each } z \in E\}$. Since $sp_{\mathbf{C}}\{\rho_\lambda^{1/2}(\psi, x) =: \phi(x) | \psi \in l_{2,b}\}$ is dense in $L^2(l_2, \lambda, \mathbf{C})$, then $sp_{\mathbf{C}}\{\rho_\mu^{1/2} =: \phi(x) | \psi \in H_b\}$ is dense in $L^2(H, \mu, \mathbf{C})$. Repeating the proof of theorem 3.8 for these groups we get that their regular unitary representations are irreducible. That is representations T of $l_{2,b}$, H_b, $c_{0,b}$ and $B(c_{0,b}, 0, 1)$ in the

corresponding spaces $L^2(l_2, \lambda, \mathbf{C})$, $L^2(H, \mu, \mathbf{C})$, $L^2(c_0, \eta, \mathbf{C})$ and $L^2(B, \nu, \mathbf{C})$ (the first case was also considered more generally for additive groups of locally convex spaces in [1, 10]). These groups are Banach-Lie and Abelian, moreover, each one-parameter subgroup of H_b and of $B(c_{0,b}, 0, 1)$ over $\mathbf{R}$ and $\mathbf{L}$ respectively is compact. The projections of μ and ν on one-parameter subgroups are equivalent with the Haar measures on them. Certainly, H_b and $B(c_{0,b}, 0, 1)$ are not locally compact, since T_eH_b and $T_eB(c_{0,b}, 0, 1)$ are infinite-dimensional Banach spaces over $\mathbf{R}$ and $\mathbf{L}$ respectively.

3.18. Note. Regular representations U_m of the groups $l_{2,b}$, H_b, $c_{0,b}$ or $B(c_{0,b}, 0, 1)$ from the proof of theorem 3.17 in the space $L^2(\tilde{\Gamma}_X, P_m, \mathbf{C})$ with $X = l_2$, H, c_0 or B and $m = \lambda$, μ, η or ν respectively are reducible, since $D^nL_h = 0$ for each $n > 1$ and f_0 is not cyclic for U_m (see the proof of theorem 3.9).

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